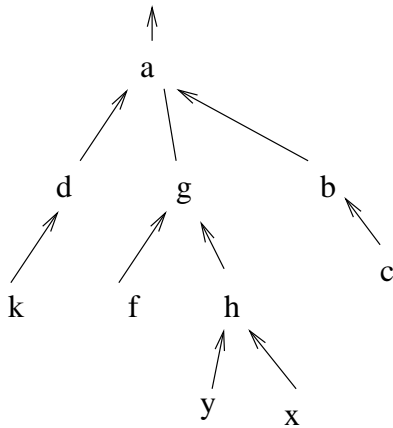


Homework 6 (Cpt S 223)
Due Date: December 7, 2011
Total points: 27

1. (12 points)

We are given two programs A and B that use two different implementations of the union-find data structure. Program A applies path compression when it performs each $find()$ operation; whereas Program B *does not* apply path compression for its $find()$ s.

Both programs start off with the same initial union-find data structure shown below¹:



And both programs perform the same sequence of $find()$ operations (in the specified order):

$find(y), find(x), find(k), find(y), find(x), find(h)$

Calculate the number of steps each of the above $find()$ operations takes to climb from the element being searched to the root node. For example, the number of such steps if one were to perform a $find(d)$ on the above shown initial tree will be 1 (under both programs). Give your answer by filling the number of steps for each $find()$ operation in the table below:

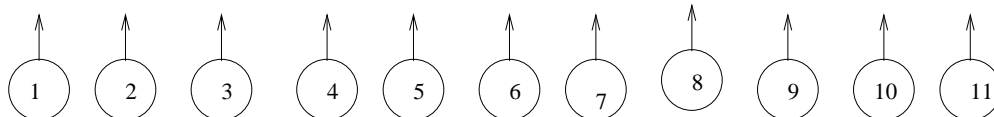
¹note that this union-find data structure contains only one tree in its forest and element a is its root.

	Program A (w/ path compression)	Program B (w/o path compression)
1. $find(y)$		
2. $find(x)$		
3. $find(k)$		
4. $find(y)$		
5. $find(x)$		
6. $find(h)$		
Total		

2. (15 points)

Starting with the union-find data structure shown below², show the *sequence of* union-find data structures that result from applying the following operations (in that order):

$union(1, 2)$, $union(3, 4)$, $union(4, 5)$, $union(6, 8)$, $union(5, 8)$, $union(1, 6)$, $union(7, 9)$, $union(10, 11)$, $union(11, 9)$, $union(1, 11)$.



Answer this question for each of the three following parts separately:

- The *unions* are performed by height (same as union-by-rank) and finds are simple;
- The *unions* are performed by size and finds are simple;
- The *unions* are performed by height and finds use path compression.

Note: There could be more than one correct answer within each part. You just need to give one.

²You can see that this initial data structure contains 12 trees in its forest.