Lemma: Binomial Heap insertion sequence of n elements takes O(n) time

Proof:

Idea = For every new insertion, at most 2^{LSZ} - 1 comparisons are made.

Bit Positions: 4 3 2 1 0

LSZ case:

\[
\begin{array}{c|c}
\text{LSZ case} & \text{take time} \\
\hline
\text{0} & 1 \\
\text{0 0 0 0} & 2 \\
\text{0 0 0 1} & 3 \\
\text{0 0 1 0} & 4 \\
\text{0 1 0 0} & 5 \\
\text{0 1 0 1} & 6 \\
\text{0 1 1 0} & 7 \\
\text{0 1 1 1} & 8 \\
\end{array}
\]

= Total (amortized) cost of n successive inserts

\[
\sum_{i=1}^{n} \frac{n \cdot x^{i}}{2^{i}} = \frac{n \cdot x}{2} + \frac{n \cdot x^{2}}{4} + \cdots + \frac{n \cdot x^{k}}{2^{k}}
\]

where k = \log n

= \sum_{i=1}^{k} \frac{n \cdot x^{i}}{2^{i}}

= n \sum_{i=1}^{k} \frac{x^{i}}{2^{i}} = 2n

= Total amortized cost of n successive inserts

\[= O(n)\]

= Amortized cost of each insert operation = O(1)