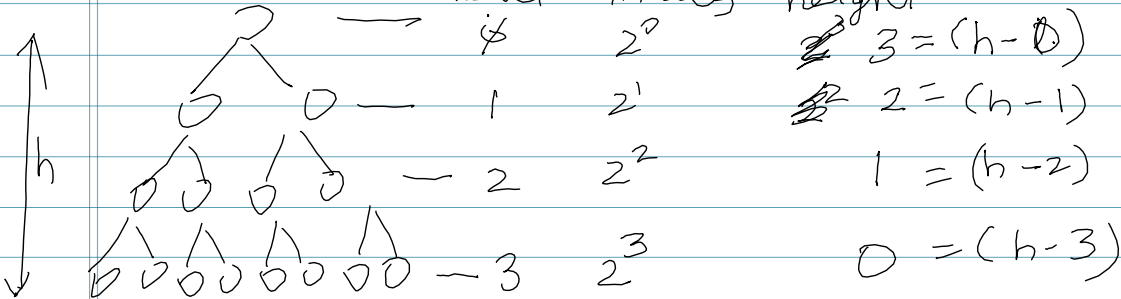


Build Heap analysis

Thm: Runtime = $O(\text{sum of Heights of all int nodes})$
 $\equiv O(\text{sum of heights of all nodes})$ (incl. leaves)
 $\Rightarrow = O(n)$

Proof:

~~Runtime~~ Sum of heights of all nodes = sum of heights of nodes at each level



$\sum_{i=0}^h$ heights of all nodes

$$= \sum_{i=0}^h (h-i) \times 2^i$$

$$= \sum_{i=0}^h h 2^i - \sum_{i=0}^h i 2^i$$

$$= h \sum_{i=0}^h 2^i - \sum_{i=0}^h i 2^i$$

$$= h(2^{h+1} - 1) - S$$

$$= h(2^{h+1}) - h - (h-1)2^{h+1} + 2$$

$$= 1 \cdot 2^{h+1} - h + 2$$

$$= 2^{h+1} - h + 2 \quad \checkmark$$

As $h = \lg_2 N$

$$\Rightarrow \sum \text{heights} = 2^{h+1} - h + 2$$

$$= 2N - \lg N + 2 = O(N)$$

Let $S = \sum_{i=0}^h i 2^i$

$\rightarrow S = 1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + 4 \cdot 2^4 + \dots + h 2^h$

$2S = 1 \cdot 2^2 + 2 \cdot 2^3 + 3 \cdot 2^4 + 4 \cdot 2^5 + \dots + h 2^h$

$2S - S = [1 \cdot 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^h] + h 2^{h+1}$

$\Rightarrow S = [2^{h+1} - 1] + h 2^{h+1}$

$= (h-1) 2^{h+1} - 2$