Homework 4

Cpt S 317, Spring 2009

Due Date: March 6, 2009

Total points: 63

1. (10 points)

   a) If L is a language and a is a symbol, then L/a, the *quotient* of L and a, is the set of strings w such that $wa \in L$. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\epsilon, ba\}$. Prove that if L is regular, so is L/a.

      Hint: Start with a DFA for L and consider the set of accepting states.

   b) Now, define $a \backslash L$ to be the set of strings w such that $aw \in L$. For example, $L = \{a, aab, baa\}$, then the language $a \backslash L = \{\epsilon, ab\}$. Prove that if L is regular, so is $a \backslash L$.

      Hint: Try to build your proof by reusing previously proven results. For example, you can think of reusing known facts about the closure properties of regular languages and also the result from part (a).

2. (5 points)

   Let L be any *finite language with finite length strings*. Is it possible to satisfy the pumping lemma for regular languages for all such Ls? In other words, are all finite languages regular? Justify your answer.

   Hint: Read the statement of the pumping lemma for regular languages carefully.

3. (8 points)

   Exercise 4.4.2.

4. (20 points)

Design a CFG for the following languages:

a) The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$ — i.e., the set of strings over alphabet $\{a, b, c\}$ s.t., in each string is a's followed by b's followed by c's, and there are either unequal number of a's and b's or unequal number of b's and c's.

b) The set of all strings over alphabet $\{a, b\}$ such that the number of a's is *at least* as many as the number of b's.

c) The set of all strings over alphabet $\{a, b\}$ that are of odd length.

(Note: this language is also regular.)

d) The set of all strings of a's and b's that are *not* of the form ww^R.

5. (10 points)

Exercise 5.1.2: parts b and c.

6. (10 points)

Exercise 5.1.7: part a.