1. (18 points) Let:

\[ L_1 = \{ a^n b^{2m} c^m | m, n \geq 1 \} \]
\[ L_2 = \{ a^n b^m c^{2m} | m, n \geq 1 \} \]

a) Give CFGs for \( L_1 \) and \( L_2 \).

b) Is \( L_1 \cap L_2 \) a CFL? Justify your answer.

c) Using the CFG designed for \( L_1 \) as a template, design another CFG for the language (denoted as \( L_{\text{pref}} \)) of all strings that are prefixes of the strings in \( L_1 \) — i.e.,

\[ L_{\text{pref}} = \{ x | x \text{ is a prefix of a string in } L_1 \} \]

Note, by this definition, each string in \( L_1 \) will generate multiple strings in \( L_{\text{pref}} \). For example, the string \( abbcc \) which is in \( L_1 \) will generate the following list of strings in \( L_{\text{pref}} \): \( \{ \epsilon, a, ab, abb, abbc, abbcc \} \).
2. (7 points) Design a Turing Machine for the language of strings of the form: 

\(a^n b^n c^m\), where \(m \geq n\), and \(m, n \geq 0\).

Answer the question either in the form of a state machine (preferred), or provide an English language step-by-step pseudocode describing the main logic of your TM design.

3. (7 points) Exercise 8.2.3 part a. Give the answer in the form of a state diagram.

4. (6 points) Exercise 8.2.5 part b. Give the answer in the form of a state diagram.