#4 $L = \{ a^i b^j c^k \mid i < j < k \}$

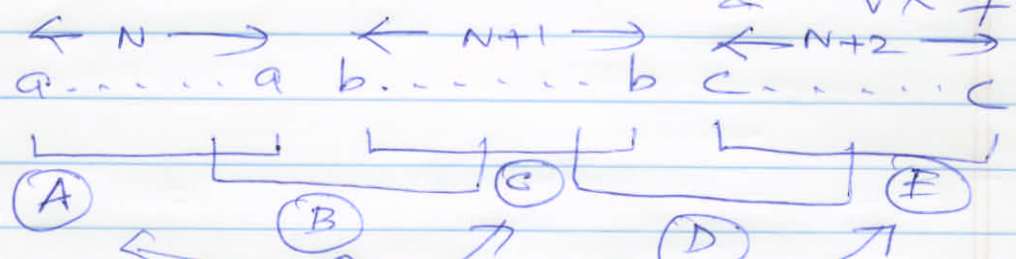
Proof: By contradiction, let L be a CFL

⇒ Let $N \leftarrow$ P/L const

Let $z = a^N b^{N+1} c^{N+2}$, $z \in L$, $|z| \geq N$

We can split $z = uvwxy$, s.t $|vwx| \leq N$ & $vx \neq \varepsilon$

z:



Diff. ways to slide vwx on z

CASE (A): vwx contains only a's

↓ $k = 0$ ⇒ reducing #a's does not violate L

⇒ $uv^0wx^0y \in L$ (not a contradiction)

↑ $k = 2$ ⇒ #a's ≥ #b's

⇒ $uv^2wx^2y \notin L$

CASE (B): a a . . . a b b . . . b

$|v \quad w \quad x|$

Subcase i) v is ε ⇒ x contains at least one b

Subcase ii) v is not ε ⇒ ↑ $k=2$: $uv^2wx^2y \notin L$

⇒ v contains at least one a

(or) ⇒ if v contains only a's ⇒ ↑ $k=2$ $uv^2wx^2y \notin L$

(or) ⇒ if v contains a's and b's ⇒ ↑ $k=2$, $uv^2wx^2y \notin L$

(Proof contd...)

Case (C): vwx contains only b's (atmost N b's)

$a a \ldots a \; b b \ldots b \; c c \ldots c$

$|\leftarrow vwx \rightarrow|$

Case (C) similar to case (A)

$\Rightarrow \uparrow k=2$: increased b's without changing #c's

$\Rightarrow uv^2wx^2y \notin L$

Case (D): vwx contains $\ldots \underbrace{b \ldots b}_{v \; w \; x} c \ldots c \ldots$

Subcases: i) v is ε $\Rightarrow$ x should contain atleast one c

$\Rightarrow \downarrow k=0$ will reduce c's without reducing b's (or) will reduce both b's & c's without reducing a's

$\Rightarrow uv^0wx^0y \notin L$

Subcases: ii) v is not ε

$\Rightarrow$ v should contain atleast one b

$\Rightarrow \downarrow k=0$ will reduce b's without changing #a's

$\Rightarrow uv^0wx^0y \notin L$

Case (E): vwx contains only c's ($\leq N$ c's)

$a a \ldots a \; b b \ldots b \; c c \ldots c$

$|\leftarrow vwx \rightarrow|$

$\downarrow k=0$: will reduce c's without changing b's

$\Rightarrow uv^0wx^0y \notin L$

$\therefore L$ is not a CFL. □