

Input

$$\Sigma = \{0, 1\}$$

$$\text{Let } L = \{ww^R \mid w \in \{0, 1\}^*\}$$

$$\begin{cases} S(0) = \{a^n b^n \mid n \geq 1\} \rightarrow \text{CFL} \checkmark \\ S(1) = \{cc, dd\} \rightarrow \text{CFL} \checkmark \end{cases}$$

To show: $S(L)$ is a CFL

Approach: to build a grammar for $S(L)$

$$G : [S \Rightarrow 0S_0 \mid 1S_1 \mid \epsilon] \Rightarrow \text{CFG for } L$$

$$G_0 : [S_0 \Rightarrow aS_0b \mid ab] \Rightarrow \text{CFG for } S(0)$$

$$G_1 : [S_1 \Rightarrow cc \mid dd] \Rightarrow \text{CFG for } S(1)$$

G' for $S(L)$:

$$G' : S \Rightarrow \underset{S_0}{\cancel{0}} S \underset{S_0}{\cancel{0}} \mid \underset{S_1}{\cancel{1}} S \underset{S_1}{\cancel{1}} \mid \epsilon$$

$$G' \rightarrow \begin{cases} S \Rightarrow S_0 S S_0 \mid S_1 S S_1 \mid \epsilon \\ S_0 \Rightarrow a S_0 b \mid ab \\ S_1 \Rightarrow cc \mid dd \end{cases}$$