Conventional amplitude modulation (AM) radio uses carrier frequencies spaced 10 kHz apart, with 10 kHz signal bandwidth. To demodulate, one needs a narrow band-pass filter, centered at the intermediate frequency of 455 kHz. This example develops the design of a second-order band-pass filter and finds the resulting frequency response.

Problem: Design a high-Q, narrow-band band-pass filter with
  a) center frequency 455,000 Hz;
  b) bandwidth 10,000 Hz.
Determine the attenuation 10 kHz, 20 kHz, 30 kHz, and 40 kHz away from the filter center frequency.

An ideal “brick-wall” band-pass filter frequency response is shown below.

[Diagram of ideal "brick-wall" bandpass filter]
A series RLC band-pass filter has the transfer function

\[ H(s) = \frac{R}{Ls} \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\beta s}{s^2 + \beta s + \omega_0^2} . \]

Suppose \( C = 10 \text{ pF} \). Then the required inductance is found as

\[ \omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi \times 455,000)^2 (1 \times 10^{-11})} = 0.0122 \text{ H} . \]

The required resistance is determined as follows.

\[ \beta = \frac{R}{L} \Rightarrow R = L\beta = (0.0122)(2\pi \times 10,000) = 766.5 \text{ } \Omega \]

Design: \( R = 766.5 \text{ } \Omega, \ L = 0.0122 \text{ } H, \ C = 10 \text{ } \text{pF} \)

Quality Factor: \( Q = \omega_0 / \beta = 45.5 \); Poles: \( s_{1,2} = 10^4 \pi (-1 \pm 90.99j) \)

(The poles are complex-valued. Note that the magnitude of the imaginary part is much larger than the magnitude of the real part.)

Cutoff frequencies:

\[ \omega_{c_1} = -0.5\beta + \sqrt{(0.5\beta)^2 + \omega_0^2} = 2\pi \times 4.5003 \times 10^5 \text{ rad/sec}, \]

\[ \omega_{c_2} = 0.5\beta + \sqrt{(0.5\beta)^2 + \omega_0^2} = 2\pi \times 4.6003 \times 10^5 \text{ rad/sec}. \]

Matlab code: % Set gain K = 1 for convenience

```matlab
>> A=[1 2*pi*10000 (2*pi*455000)^2];
>> B=[2*pi*10000 0];
>> sys=tf(B,A);
>> bode(sys)
>> w=2*pi*[0:1000:100000];
>> w=w+2*pi*400000;
>> H=freqs(B,A,w);
>> plot(w/(2*pi),20*log10(abs(H)))
```
Bode plots for the second-order bandpass filter with center frequency 455 kHz and bandwidth 10 kHz. At carrier frequencies 20, 40 and 60 kHz above 455 kHz (i.e., at 2.98, 3.11 and 3.24 Mrad/s) the filter gains are -11.8, -17.8 and -22 dB, respectively.

Figure 1. Bode frequency response for narrow-band bandpass filter.
Figure 2. Filter frequency response magnitude for frequencies near 455 kHz.

Figure 3. Filter frequency response magnitude over wide range of frequencies. Slope of response far away from 455 kHz is $\pm 20$ dB/decade.