P 12.30  

[a] \[ \frac{v_o - V_{dc}}{R} + \frac{1}{L} \int_0^t v_o \, dx + C \frac{dv_o}{dt} = 0 \]

\[ \therefore v_o + \frac{R}{L} \int_0^t v_o \, dx + RC \frac{dv_o}{dt} = V_{dc} \]

[b] \[ V_o + \frac{RV_o}{L} s + RC s V_o = \frac{V_{dc}}{s} \]

\[ \therefore s L V_o + R V_o + R C L s^2 V_o = L V_{dc} \]

\[ \therefore V_o(s) = \frac{(1/RC)V_{dc}}{s^2 + (1/RC)s + (1/LC)} \]

[c] \[ i_o = \frac{1}{L} \int_0^t v_o \, dx \]

\[ I_o(s) = \frac{V_o}{sL} = \frac{V_{dc}/RLC}{s[s^2 + (1/RC)s + (1/LC)]} \]
Based on Figure P12.28:

\[ V_{Re} = 200,000 \quad \frac{1}{LC} = 10^{10} \]

After having the switch on position a for a long time, the capacitor is open and the inductor is a short circuit:

\[ I_{dc} = 0.04 \text{ A} \]

From Problem 4, we know that:

\[ I_0(s) = \frac{I_{dc} [s^2 + (1/RC)]}{\left[ s^2 + (1/RC) \right] s + (1/RC)} \]

\[ I_0(s) = \frac{0.04(s + 200,000)}{s^2 + 200,000s + 10^{10}} \]

\[ = \frac{K_1}{(s + 100,000)^2} + \frac{K_2}{s + 100,000} \]

\[ K_1 = 0.04(s + 200,000) \quad s = -100,000 \]

\[ K_2 = \left. \frac{d}{ds} \left[ 0.04(s + 200,000) \right] \right|_{s = -100,000} = 4000 \]

\[ I_0(s) = \frac{4000}{(s + 100,000)^2} + \frac{0.04}{s + 100,000} \]

**Using inverse Laplace transform:**

\[ i_o(t) = \left[ 4000e^{-100,000t} + 0.04e^{-100,000t} \right] u(t) \]

**For** \( t > 0 \)
The circuit parameters in the circuit in Fig P12.25 are \( R = 4 \text{k} \Omega \), \( L = 400 \text{mH} \), \( C = 156.25 \text{nF} \). If \( v_{dc} = 120 \text{V} \), find \( v_o(t) \) for \( t \geq 0 \).

**Solution:**

Here \( R_L = 10 \text{k} \Omega \); 

\[
\frac{1}{L_C} = 16 \times 10^6
\]

\[
\therefore \quad v_o(s) = \frac{120(s+10000)}{s+10000 + 16 \times 10^6}
\]

\[
= \frac{120(s+10000)}{(s+2000)(s+8000)}
\]

\[
= \frac{K_1}{s+2000} + \frac{K_2}{s+8000}
\]

\[
\therefore \quad K_1 = \frac{120(2000)}{8000} = 160
\]

\[
K_2 = \frac{120(8000)}{-6000} = -40
\]

\[
\therefore \quad v_o(s) = \frac{160}{s+2000} - \frac{40}{s+8000}
\]

\[
\therefore \quad v_o(t) = \left\{ \begin{array}{ll}
-2000t - 8000t & \text{for } t \geq 0
\end{array} \right\} \text{V}
\]
\[ F(s) = \frac{2s^2 + 33s^2 + 93s + 54}{s(s+1)(s+2)(s+3)} \]

\[ F(s) = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2} + \frac{k_4}{s+3} \]

\[ K_1 = \frac{2s^2 + 33s^2 + 93s + 54}{(s+1)(s+2)(s+3)} \bigg|_{s=0} = 9 \]

\[ K_2 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+2)(s+3)} \bigg|_{s=-1} = 4 \]

\[ K_3 = \frac{2s^2 + 33s^2 + 93s + 54}{s(s+1)(s+3)} \bigg|_{s=-2} = -8 \]

\[ K_4 = \frac{2s^3 + 33s^2 + 93s + 54}{s(s+1)(s+2)} \bigg|_{s=-3} = -3 \]

\[ \Rightarrow F(s) = \frac{9}{s} + \frac{4}{s+1} + \frac{-8}{s+2} + \frac{-3}{s+3} \]

\[ f(t) = \left[ 9 + 4e^{-t} - 8e^{-2t} - 3e^{-3t} \right] u(t) \]
Find \( f(t) \) for each of the following functions.

a) \( F(s) = \frac{2.8}{s^2 + 14s + 245} \)

\( F(s) = \frac{-s^5 + 52s + 445}{s(s^2 + 105 + 89)} \)

\( F(s) = \frac{14s^2 + 56s + 152}{(s+6)(s^2 + 4s + 20)} \)

\( F(s) = \frac{9}{(s+1)^2(s^2 + 105 + 89)(s^2 + 8s + 20)} \)

Solution:

\( F(s) = \frac{k_1}{s + 7 - 14j} + \frac{k_1^*}{s + 7 + 14j} \)

\[ k_1 = \frac{2.8}{s + 7 + 14j} \bigg|_{s = -7 + 14j} = -10^\circ = 10 \angle 90^\circ \]

\[ f(t) = 20 e^{-7t} \cos(14t - 90) \ u(t) \]

\[ = 20 e^{-7t} \sin(14t) \ u(t) \]

b) \( F(s) = \frac{k_1}{s} + \frac{k_2}{s + 5 - 8j} + \frac{k_2^*}{s + 5 + 8j} \)

\[ k_1 = \frac{-s^5 + 52s + 445}{s(s^2 + 8j)} \bigg|_{s = 0} = 5 \]

\[ k_2 = \frac{-s^5 + 52s + 445}{s(s^2 + 8j)} \bigg|_{s = -5 + 8j} = -3 - 2j = 3.6 \angle -146.31^\circ \]

\[ f(t) = \left( 5 + 7.2 e^{-5t} \cos(8t - 146.31^\circ) \right) u(t) \]
\( F(s) = \frac{k_1}{s+6} + \frac{k_2}{s+2} + \frac{k_3^*}{s+2+4j} \)

\[
\begin{align*}
  k_1 &= \frac{14s^2 + 56s + 152}{s^2 + 4s + 26} \bigg|_{s=-6} = 10 \\
  k_2 &= \frac{14s^2 + 56s + 152}{(3+6)(s+2+4j)^2} \bigg|_{s=-2+4j} = 2+2j = 2.83 (4.5^\circ) \\
  f(t) &= \left\{ 10e^{-6t} + 5.4e^{-4t} \cos(4t + 45^\circ) \right\} \cdot u(t)
\end{align*}
\]

(6) P 12.42 \( F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s + 8} \)

\[
\begin{align*}
  K_1 &= \frac{320}{s + 8} \bigg|_{s=0} = 40 \\
  K_2 &= \frac{d}{ds} \left[ \frac{320}{s + 8} \right] = \left[ \frac{-320}{(s + 8)^2} \right]_{s=0} = -5 \\
  K_3 &= \frac{320}{s^2} \bigg|_{s=-8} = 5 \\
  f(t) &= [40t - 5 + 5e^{-8t}]u(t)
\end{align*}
\]

(c) \( F(s) = \frac{K_1}{(s+1)^2} + \frac{K_2}{s+1} + \frac{K_3}{s+3-j4} + \frac{K_3^*}{s+3+j4} \)

\[
\begin{align*}
  K_1 &= \frac{60(s+5)}{s^2 + 6s + 25} \bigg|_{s=-1} = 12 \\
  K_2 &= \frac{d}{ds} \left[ \frac{60(s+5)}{s^2 + 6s + 25} \right] = \left[ \frac{60}{s^2 + 6s + 25} - \frac{60(s+5)(2s+6)}{(s^2 + 6s + 25)^2} \right]_{s=-1} = 0.6 \\
  K_3 &= \frac{60(s+5)}{(s+1)(s+3+j4)} \bigg|_{s=-3+j4} = 1.68/100.305^\circ \\
  f(t) &= [12te^{-t} + 0.6e^{-t} + 3.35e^{-3t} \cos(4t + 100.305^\circ)]u(t)
\end{align*}
\]
\[ 12.19 \]

\[ SV(s) = \frac{V_c}{sRC} \]

\[ \lim_{s \to 0} SV(s) = 0 \Rightarrow V_c(0) = 0 \]

\[ \lim_{s \to \infty} SV(s) = 0 \Rightarrow V_c(\infty) = 0 \]

\[ \frac{1}{s^2 + \frac{1}{RC}s + \frac{1}{2C}} \]

\[ \lim_{s \to 0} \frac{V_c}{s} = \frac{V_{dc}}{RC} \Rightarrow I_c(\infty) = \frac{V_{dc}}{RC} \]

\[ \lim_{s \to \infty} \frac{V_c}{s} = 0 \Rightarrow I_c(0) = 0 \]

\[ \frac{12.50, b) and d) only}{\]

\[ sF(s) = \frac{20s^2 + 141s + 315}{s^2 + 10s + 21} \]

\[ \lim_{s \to 0} sF(s) = 15; \quad \therefore f(\infty) = 15 \]

\[ \lim_{s \to \infty} sF(s) = 20, \quad \therefore f(0^+) = 20 \]

\[ \frac{[b]}{\}

\[ sF(s) = \frac{2s^3 + 33s^2 + 93s + 54}{(s + 1)(s^2 + 5s + 6)} \]

\[ \lim_{s \to 0} sF(s) = 9, \quad \therefore f(\infty) = 9 \]

\[ \lim_{s \to \infty} sF(s) = 2, \quad \therefore f(0^+) = 2 \]

\[ \frac{[d]}{\}

\[ 12.57, b) & c) only {\]

\[ sF(s) = \frac{-s^2 + 52s + 445}{s^2 + 10s + 89} \]

\[ \lim_{s \to 0} sF(s) = 5, \quad \therefore f(\infty) = 5 \]

\[ \lim_{s \to \infty} sF(s) = -1, \quad \therefore f(0^+) = -1 \]

\[ \frac{[b]}{\}

\[ sF(s) = \frac{14s^3 + 56s^2 + 152s}{(s + 6)(s^2 + 4s + 20)} \]

\[ \lim_{s \to 0} sF(s) = 0, \quad \therefore f(\infty) = 0 \]

\[ \lim_{s \to \infty} sF(s) = 14, \quad \therefore f(0^+) = 14 \]

\[ \frac{[c]}{\}