Consider the circuit shown below. The switch has been in position $a$ for a long time. At time $t = 0$ the switch moves to position $b$.

i) Find voltage $v_0(t)$ for $t \geq 0$.

ii) Find a state variable representation for the circuit for $t \geq 0$. Specify all required initial conditions.

Solution.
1. When the switch is in position $a$, find the initial conditions for capacitor voltage and inductor current.

2. When the switch moves to position $b$, let $R$ be the parallel combination of the 24 k and 12 k resistors, and derive the differential equation for the capacitor voltage, $v_C(t)$. Solve the differential equation.

3. Solve for the voltage $v_0(t)$ in terms of $v_C(t)$.

4. Define state variables from the differential equation for $v_C(t)$. Solve for voltage $v_0(t)$ in terms of the state variables. Use Matlab to solve. Plot the solution.
\[
\frac{d^2 v_C}{dt^2} + \frac{R}{L} \frac{dv_C}{dt} + \frac{1}{LC} v_C = \frac{1}{LC} v_i(t), \quad v_C(0^+) = -16, \quad \frac{dv_C(0^+)}{dt} = \frac{0.003}{C}
\]

\[
A = \begin{bmatrix}
0 & 1 \\
-1/L & -R/L \\
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
1/LC \\
\end{bmatrix}, \quad C = [1 \ R \ C], \quad D = [0]
\]

```matlab
function state_variables_example3
% Example of state variables
% L = 0.1 H, R = 560 ohm, Cap = 10^(-7) F
L=0.2;Cap=8e-9;R=8000;
A=[0 1;-1/(L*Cap) -R/L];B=[0;1/(L*Cap)];C=[1 R*Cap];D=[0];
sys1=ss(A,B,C,D); % Form sys1 as the state space model
x0=[-16;0.003/Cap];
t=[0:0.000001:0.00025];
vi=20*ones(1,length(t)); % unit step input
[y,x]=lsim(sys1,vi,t,x0);
figure(1)
subplot(2,1,1)
pplot(t,y(:,1))
xxlabel('Time, t, sec')
yylabel('Voltage, v_0(t), V')
title('Voltage, v_0(t), Example Lecture 4')
subplot(2,1,2)
pplot(t,x(:,1))
xxlabel('Time, t, sec')
yylabel('Voltage, v_C(t), V')
title('Capacitor Voltage, v_C(t), Example Lecture 4')
```