(Taken from Assessment Exercise 16.6 in text.) The periodic square wave shown below is applied to the circuit. $R = 10 \, \text{k}\Omega$, $L=20 \, \text{mH}$, $C=20 \, \text{nF}$.

a) Derive the first four non-zero terms in the Fourier series of the steady-state output voltage, if $V_m = 210\pi \, \text{V}$ and the period of the input square wave is $0.2\pi \, \text{ms}$.

b) Which harmonic dominates the output voltage? Explain why.

c) Determine the percentage of total power in the $5^{th}$ harmonic of the input and output signals.

Period $T = 0.2\pi \, \text{ms}$, so $\omega_0 = \frac{2\pi}{T} = 10^4 \, \text{rad/sec}$.

Square wave Fourier Series coefficients: $a_0 = 0$, $b_n = 0$, $a_n = \frac{840}{n} \sin\left(\frac{n\pi}{2}\right)$.

Fourier series is $v_i(t) = \sum_{n=1}^\infty a_n \cos(n\omega_0 t)$

Hence, the first four non-zero terms correspond to $n = 1, 3, 5, 7$, and frequencies $10,000$, $30,000$, $50,000$ and $70,000 \, \text{rad/sec}$.

Band-pass filter: $H(s) = \frac{s}{s^2 + \frac{s}{RC} + \frac{1}{LC}} = \frac{\beta s}{s^2 + \beta s + \omega^2} = \frac{5000s}{s^2 + 5000s + 50000^2}$.

Fourier series of filter output: $v_0(t) = \sum_{n=1}^\infty a_n |H(jn\omega_0)| \cos(n\omega_0 t + \theta_n)$, $\theta_n = \angle H(jn\omega_0)$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n\omega_0$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>840</td>
<td>-280</td>
<td>168</td>
<td>-120</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>H(jn\omega_0)</td>
<td>$</td>
<td>0.0208</td>
<td>0.0933</td>
<td>1</td>
</tr>
<tr>
<td>$\text{phase}(H(jn\omega_0))$</td>
<td>88.8°</td>
<td>84.64°</td>
<td>0°</td>
<td>-81.7°</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>a_n</td>
<td>\times</td>
<td>H(jn\omega_0)</td>
<td>$</td>
<td>17.5</td>
</tr>
</tbody>
</table>
First four non-zero term truncated Fourier series for input, \( v_i(t) \), and output, \( v_o(t) \).

Determine the percentage of the total power in the 5\(^{th}\) harmonic. For the input signal,

\[
P = \frac{1}{T} \int_0^T v_i^2(t) dt = V_m^2 = (210\pi)^2.
\]

The percentage of the total power in the 5\(^{th}\) harmonic is then

\[
\frac{0.5 \times (a_5^2 + b_5^2)}{P} \times 100\% = 3.24\%.
\]

Finding the percentage of power in the 5\(^{th}\) harmonic of the filter output is more challenging, since the signal power cannot be simply calculated. Instead, use

\[
P_{v_o} = 0.5 \times \sum_{n=odd} a_n^2 |H(jn\omega_0)|^2 = 14,804,
\]

adding up as many terms as necessary until the sum converges. Then the percentage power in the 5\(^{th}\) harmonic is

\[
\frac{0.5 \times a_5^2 \times |H(j5\omega_0)|^2}{P_{v_o}} \times 100\% = \frac{0.5 \times a_5^2}{P_{v_o}} \times 100\% = 95.3\%
\]

since \( H(j5\omega_0) = 1 \).