Problem 2: \[ V = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0 \Rightarrow V = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \cdot V_0 \]
\[ V = V_0 = 0 \Rightarrow V = -\left( \frac{R_3}{R_1} V_1 + \frac{R_2}{R_2} V_2 \right) \]

i) To have \( V = -\frac{V_1}{R_1} \), set \( R_3 = R_1 \) and \( R_2 = R_2 \).

ii) To have \( V = -\frac{V_2}{R_2} \), set \( R_3 = R_1 \), \( R_2 = \frac{1}{2} R_3 \).

Problem 7: \[ H(s) = -\frac{R_2 + \frac{1}{sC_2}}{R_1 + \frac{1}{sC_1}} = -\frac{R_2}{R_1} \left( \frac{1}{s + \frac{1}{C_1 R_1}} \right) \]

DC gain \( \frac{V_o}{V_i} = \frac{C_1}{C_2} \)

High freq gain \( \frac{V_o}{V_i} \) \( \text{dB} \)

6 dB \( \Rightarrow \) factor of \( 2 \), so set \( \frac{C_1}{C_2} = 2 \).

High freq gain \( = 2 \) dB \( \Rightarrow \) \( \frac{C_1}{C_2} = 10 \)

Output \( V_o = \frac{1}{R_2} = 20 \Omega \)

Input \( V_i = \frac{1}{R_1} = \frac{1}{2} \Omega \)

Here, \( C_1 = 1 \mu F \) \( \Rightarrow \) \( C = C_2 = 0.5 \mu F \)

\( \omega_p = \frac{1}{R_C} = 2000 \Omega \) \( \Rightarrow \) \( R = \frac{2000}{0.5 \times 10^6} = 159.2 \Omega \)

\( \omega_2 = \frac{1}{R_2} = 2000 \Omega \) \( \Rightarrow \) \( R = \frac{2000}{0.5 \times 10^6} = 318.3 \Omega \)

As a check, \( \frac{1}{R_2} = 159.7 \Omega \)

Problem 8: Solution

Omitted

Problem 5: \( \omega_c = 10 \text{kHz}, \text{low freq gain} = 6 \text{ dB} \)

\[ H(s) = -\frac{R_2 + \frac{1}{sC_2}}{R_1 + \frac{1}{sC_1}} = -\frac{R_2}{R_1} \left( \frac{1}{s + \frac{1}{C_1 R_1}} \right) \]

Set \( \omega_c = \frac{1}{R_C} = \frac{1}{R_1} \), so \( C = 1 \mu F \), \( R = \frac{1}{2\pi \times 10^4} = \frac{1}{159.2} = 159.2 \Omega \)

The low freq gain \( = \frac{R_2}{R_1} \) \( \text{dB} \)

Problem 6: Design for high freq gain \( A \) \( \text{dB} \) (\( \text{gain} = 4 \))

Cut off freq \( \omega_c = \frac{1}{RC} \)

\[ H(s) = -\frac{R_2}{R_1} \left( \frac{1}{s + \frac{1}{C_1 R_1}} \right) \]

\( \omega_c = 1 \mu F \) \( \text{high freq gain} = 6 \text{ dB} \)

\( \frac{R_2}{R_1} = A \) \( \text{(for 6 dB high freq gain)} \)

\( R_C = 4 \Omega \)
6. Problem 9, Low range \([50, 160] \text{ kHz}\), \(C = 2.2 \text{nF}\) caps.

\[ H(f) = \left( \frac{-R_{21} R_{22}}{R_{22} S + 1} \right) \left( \frac{-R_{21} S}{R_{11} S + 1} \right) \]

**Low-pass Section**

- Set \( \omega_c = 2 \times 10^{4} \text{ rad/s} \), \( \text{sec} = \frac{1}{R_{21} C} = \text{low-pass cutoff} \)
- \( R_{21} = \frac{1}{(2 \times 10^{4})(2.2 \times 10^{-9})^2} = 452 \Omega \)

**High-pass Section**

- Set \( \omega_c = 2 \times 10^{5} \text{ rad/s} \), \( \text{sec} = \frac{1}{R_{11} C} = \text{high-pass cutoff} \)
- \( R_{11} = \frac{1}{(2 \times 10^{5})(2.2 \times 10^{-9})^2} = 6028.6 \Omega \)

To have unity gain at the center frequency,
- Set \( R_{21} = R_{11} = 452 \Omega \), \( R = X = 6028.6 \Omega \)

To have 20 dB gain, set \( R_{21} = R_{11} = 6028.6 \Omega \)

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7. Design wideband bandreject filter for

High range \([5, 100] \text{ kHz}\). Use \( C = 1 \text{F}\) caps.

From notes, \( \omega_c = 2 \times 10^4 \text{ rad/s} \), \( \text{sec} = \frac{1}{R_{21} C} \)

- \( R_{21} = \frac{1}{(2 \times 10^4)(2.2 \times 10^{-9})^2} = 72.3 \Omega \)

Set \( \omega_c = 2 \times 10^5 \text{ rad/s} \), \( \text{sec} = \frac{1}{R_{11} C} \)

- \( R_{11} = \frac{1}{(2 \times 10^5)(2.2 \times 10^{-9})^2} = 7.98 \Omega \)

Set \( R = R_1 = \text{for each section} \).
(c) (prob. 11) 

From notes, \( H(s) = \frac{1}{s^2 + 2s + \frac{1}{2} + \frac{1}{2} s^2} \)

Set \( R_2 = 1 \text{k} \)
\[ \frac{C_1}{C_2} \text{ and } \frac{b_1}{b_2} = \frac{2}{1} \text{. New design a 2nd order Butterworth } \]

Then \( H(s) = \frac{1}{s^2 + \frac{1}{2} s + 1} = \frac{1}{s^2 + b_1 s + 1} \Rightarrow b_1 = \sqrt{2} \)
\[ \Rightarrow b_1 = \sqrt{2} \Rightarrow C_1 = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow C_2 = \frac{1}{\sqrt{2}} \]

Now scaling: \( k_f = \frac{1}{k_m} = \frac{2}{\sqrt{2}} \)

Use \( R' = 10 \text{k} \) resistors \( \Rightarrow k_m = \frac{R'}{R} = 10 \)

Then \( R = 100 \text{k} \)
\[ C_1 = \frac{C_1}{k_f} = \frac{\sqrt{2}}{\frac{2}{\sqrt{2}}} = 225 \text{ pF} \]
\[ C_2 = \frac{C_2}{k_m} = \frac{\frac{1}{\sqrt{2}}}{10} = 112.5 \text{ pF} \]

(9) prob. 12, design for cutoff \( f_c = 200 \text{ Hz} \)

From notes, the 2nd order high-pass active filter transfer function is
\[ H(s) = \frac{\frac{2}{s^2 + \frac{2}{s} + \frac{1}{2} s^2} + \frac{1}{s^2 + b_1 s + 1}}{s^2 + b_2 s + 1} \]

Letting \( C = 1 \text{F}, \) \( R_2 = 1 \text{K}, \) \( b_1 = 2 \) \( \text{normalized} \)

Using Butterworth high-pass \( f_c = 200 \text{ Hz} \), so \( H(s) = 1 \)

For cutoff \( f_c = 1 \text{kHz} \) \( \Rightarrow \frac{\frac{2}{s}}{R} = \sqrt{2} \Rightarrow R = \sqrt{2} R_2 = \sqrt{2} \Rightarrow R_2 = \sqrt{2} \)
\[ k_f = \frac{1}{k_m} = \frac{1}{k_m} \]

Using 10 \text{nF} caps, \( C' = \frac{C}{k_f} = \frac{10^{-9}}{\frac{1}{k_m}} \Rightarrow k_m = 79577.17 \Rightarrow R_1 = k_m R_1 = \frac{k_m}{\sqrt{2}} = 5627 \text{k} \)
\[ R_2 = \frac{1}{k_m} R_2 = \frac{1}{\sqrt{2}} k_m = 112.54 \text{k} \]
Design an active Butterworth 3rd order filter with a cutoff frequency 15 kHz.

From notes:

\[ H(s) = \frac{1}{(s + \frac{1}{R_2C_2})(s^2 + \frac{2}{R_2C_2}s + \frac{1}{R_2C_2})} \]

Example: design normalized transfer function for \( \omega_c = 1 \) Hz.

Set \( R_2 = R_1 = 15 \), \( C_1 = 1 \), \( R = 1 \), \( C_1, C_2 = 1 \), \( b = \frac{2}{R_2} \).

Then \( H(s) = \frac{1}{(s + 1)(s^2 + \frac{2}{R_2C_2}s + \frac{1}{R_2C_2})} \) so need \( b = \frac{1}{R_2} \) for 3rd order Butterworth filter.

\[ b_1 = 1 = \frac{2}{R_2} \Rightarrow C_1 = 2F \Rightarrow C_1 = \frac{1}{2F} = \frac{1}{2} \]

For scaling:

\[ k = \frac{R_1'}{R_2} = \frac{15}{15} = 1 \]

Then \( R_1' = 15 \times 1 = 15 \).

Design as \( R_1 = R_2 = R = 1 \Omega \).

1st section:

\[ C_1' = \frac{2F}{k_1} = \frac{2F}{15} = 0.01333 F \]

2nd order section:

\[ C_2' = \frac{2F}{2C_1'} = 0.5305 F \]

Design active high-pass Butterworth filter, \( \omega_c = 10 \) kHz.

Use two 2nd order high-pass sections so from prob 12 note:

\[ H(s) = \frac{s^2 + \frac{2}{R_2C_2}s + \frac{1}{R_2C_2}}{(s + \frac{1}{R_2C_2})(s^2 + \frac{2}{R_2C_2}s + \frac{1}{R_2C_2})} \]

Use \( C = 1F \).

Use 4th order Butterworth from Table 15.1

\[ R_2 = 1, R_3^2 = 1 \]

\[ b_1 = 0.965 = \frac{2}{R_2} \Rightarrow R_2 = 2.054 \]

\[ R_2, R_1 = 1 \Rightarrow R_1 = \frac{1}{R_2} = 0.5 \]

\[ R_3^2 = 1 \Rightarrow R_3 = \frac{1}{R_2} = 0.5 \]
The frequency & magnitude scaling:
\[ k_f = \frac{\omega'_c}{\omega_c} = \frac{10}{10} = 1 \text{ or } 10^4 \rightarrow k_m = \frac{C}{\omega_c C'} = \frac{1}{(10^4/10^5)} = 1 \text{ or } 10^4. \]

\[ C' = 10 \text{ nF} - \text{all caps.} \]

\[ R'_1 = k_m R_1 = 382.5 \Omega \quad R'_2 = k_m R_2 = 261.48 \Omega \]

All circuits are as in on-line notes.