1. Consider the series RLC circuit shown below. There are two outputs of interest, the capacitor voltage, \( v(t) \), and the inductor voltage, \( v_L(t) \). Assume there is no energy stored in the inductor or capacitor at time \( t = 0 \). Derive a differential equation for the capacitor voltage, \( v(t) \). Find the characteristic equation. Find a state variable representation for the circuit, with outputs \( v(t) \) and \( v_L(t) \). Use Matlab to find, and plot, the outputs for the input \( v_i(t) = 5 \ u(t) \) V.

\[ t = 0 \]

\[ \begin{array}{c}
R \\
560 \ \Omega \\
\end{array} \quad \begin{array}{c}
L \\
100 \text{ mH} \\
\end{array} \quad \begin{array}{c}
C \\
0.1 \ \mu F \\
\end{array} \]
function state_variables_example2
  % Example of state variables for series RLC circuit
  % L = 0.1 H, R = 560 ohm, Cap = 10^(-7) F
  L=0.1; Cap=1e-7; R=560;
  A=[0 1;-1/(L*Cap) -R/L]; B=[0;1/(L*Cap)]; C=[1 0;-1 -R*Cap]; D=[0; 1];
  x0=[0;0];
  t=[0:0.00001:0.002];
  vi=5*ones(1,length(t)); % unit step input
  [y,x]=lsim(A,B,C,D,vi,t,x0);
  figure(1)
  subplot(2,1,1)
  plot(t,y(:,1))
  xlabel('Time, t, sec')
  ylabel('Voltage, v(t), V')
  title('Capacitor Voltage, v(t), and Inductor Voltage, v_L(t) for Series RLC Circuit')
  subplot(2,1,2)
  plot(t,y(:,2))
  xlabel('Time, t, sec')
  ylabel('Voltage, v_L(t), V')
2. A passive circuit has input voltage $v_i(t)$, and inductor current, $i_L(t)$, and resistor voltage, $v_R(t)$, as shown. Also of interest is the voltage, $v_o(t)$, across the series inductor-resistor configuration, as shown below.

The inductor current shown in the figure satisfies the third-order differential equation

$$\frac{d^3 i_L(t)}{dt^3} + 5 \frac{d^2 i_L(t)}{dt^2} + 6 \frac{di_L(t)}{dt} + 3i_L(t) = 7v_i(t) \text{ for } t \geq 0.$$  

Assume that there is no initial energy stored in the circuit (so all initial conditions are zero). There are two outputs of interest, $i_L(t)$ and $v_o(t)$, as labeled in the circuit.

a. Find a state variable representation for the system,

$$\mathcal{A} = A x + B v_i, \quad \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix} = y = C x + D v_i.$$ Define the state variables and determine the $A$, $B$, $C$, and $D$ matrices. (i.e., Find a state-variable representation for the differential equation, and then determine the outputs ($i_L(t)$ and $v_o(t)$) in terms of the state variables.)

b. Let the input voltage be the unit-step signal, $v_i(t) = u(t)$, and determine the steady-state values of $i_L(t)$ and $v_o(t)$. 

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The diagram shows a passive circuit with an inductor $L$, a resistor $R$, and the inductor current $i_L(t)$ and the voltages $v_i(t)$, $v_R(t)$, and $v_o(t)$.