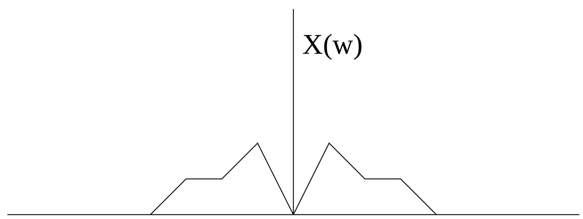
Final Exam

EE 341 Fall 2007

Closed book, two hour exam. Three (double-sided) pages of notes are allowed. Some useful formulae are appended. There are 11 problems for a total of 210 possible points.

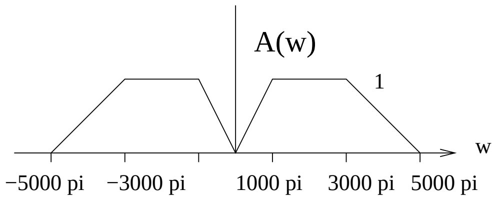
1. A signal, $x(t)$, has the spectrum shown below. An amplitude modulation signal is formed as $y(t) = A[1 + \mu x(t)] \cos(2\pi f_c t)$, where $f_c = 980$ kHz, $0 < \mu < 1$, and $\max_t |x(t)| < 1$.

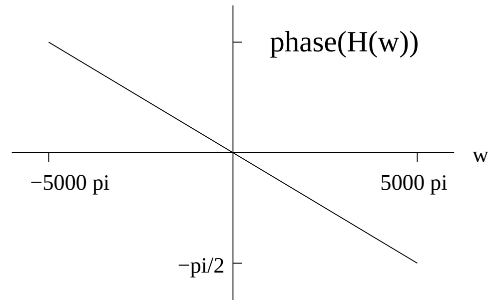


a) (5 points) What is the bandwidth of the AM signal?

b) (10 points) Find $Y(\omega)$ in terms of $X(\omega)$ and accurately sketch $Y(\omega)$, labeling relevant amplitudes and frequencies.

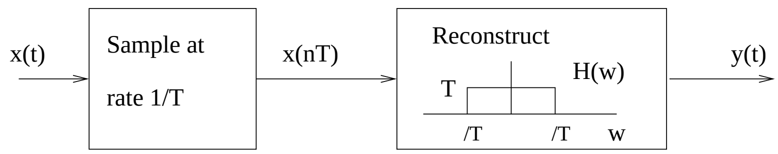
2. (10 points) A linear, time-invariant system with impulse response $h(t)$ has the frequency response $H(\omega) = A(\omega)e^{-jwt_0}$, where $A(\omega)$ is real-valued, as shown below, $t_0 = 0.0001$ sec, and the phase is also shown below.





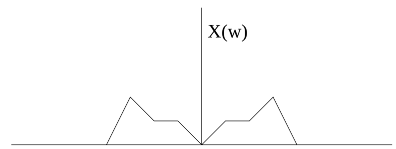
Find the system response to the input $x(t) = \cos(2\pi 250t)$.

3. A signal $x(t)$ is applied as input to a sampler, followed by an ideal lowpass filter, as shown below.



The reconstructed signal is $y(t) = \sum_{n=-\infty}^{\infty} x(nT)\text{sinc}(\frac{t-nT}{T})$.

a) (5 points) If $X(\omega)$ is as shown below, according to the sampling theorem what is the minimum sampling rate, $1/T$, to avoid aliasing distortion?



b) (10 points) Let $x(t) = 2\sin(2\pi f_1 t) + 3\cos(2\pi f_2 t)$, with $f_1 = 100$ Hz, $f_2 = 250$ Hz, and sampling rate $1/T = 400$ samples/sec. **Sketch** the spectrum at the output of the sampler and **find** $y(t)$. Simplify the expression for $y(t)$ as much as possible.

4. (15 points) Design an analog notch filter to remove the frequency 1000 Hz. Specifically, design a two-pole, two-zero notch filter. Specify the filter transfer function, $H(s)$, identify the pole and zero locations and sketch their location in the s-plane, and sketch the frequency response for your filter, labeling relevant amplitudes and frequencies.

5. (30 points) A linear, time-invariant system has the impulse response given below (two cases - one continuous-time, and one discrete-time). Determine, for each of the impulse responses, whether or not the system is i) memoryless, ii) causal, and iii) BIBO stable. **Valid justification must be given to receive credit**.

a) (15 points) Discrete-time

$$h[n] = \begin{cases} n^2, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

b) (15 points) Continuous-time

$$h(t) = \begin{cases} \frac{2}{1+|t|}, & \text{for } -1 \leq t \\ 0, & \text{otherwise} \end{cases}$$

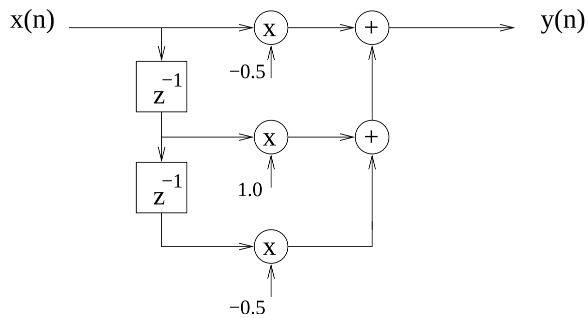
6. a. (15 points) **Find** the discrete convolution of $h(n)$ and $x(n)$ given below.

$$h(n) = (\cdots, 0, -1, 1, 1, 1, -1, 0, 0, \cdots), \quad x(n) = (\cdots, 0, 0, 1, 2, 3, 4, 3, 2, 1, 0, 0, \cdots)$$

b. (15 points) **Find** $y(t) = h(t) * x(t)$ for the signals given below. **Sketch** $y(t)$.

$$h(t) = 2e^{-(t-1)}u(t-1), \qquad x(t) = u(t) - u(t-5)$$

7. (20 points) A discrete-time, linear, time-invariant system with input $x(n)$ and output $y(n)$ has the realization shown below.



a) (5 points) Find the difference equation for the system.

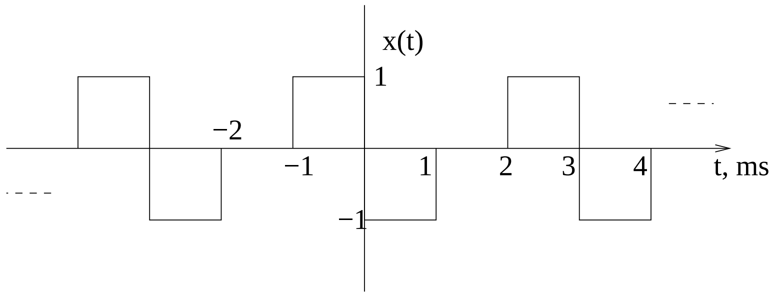
b) (5 points) Find the system impulse response, $h(n)$.

c) (5 points) Find the system output, $y(n)$, to the unit-step input,

$$x(n) = u(n) = \begin{cases} 1, & \text{for } n \geq 0; \\ 0, & \text{for } n < 0. \end{cases}$$

d) (5 points) Find the system frequency response, $H(e^{j\Omega})$, which is the discrete-time Fourier Transform of $h(n)$. Sketch $|H(e^{j\Omega})|$ over one period of the (normalized) frequency range $[-\pi, \pi]$.

8. (20 points) A periodic signal, $x(t)$, is shown below. (Note that the time axis is labeled in units of milliseconds.)



a) Determine the exponential Fourier series representation

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt/T}.$$

Specifically, determine
i) (2 points) The signal period, T.
ii) (3 points) The Fourier series coefficient c_0.
iii) (7 points) For $n \neq 0$, the Fourier series coefficients c_n.

b) (8 points) The signal $x(t)$ is passed through a linear, time-invariant filter with frequency response

$$H(\omega) = \begin{cases} e^{-j\omega t_0}, & \text{for } 1000\pi < |\omega| < 1500\pi; \\ 0, & \text{otherwise,} \end{cases}$$

where $t_0 = 0.00025$ second.

Find the filter output, $y(t)$. Simplify as much as possible.

9. (20 points) A discrete memoryless source, X, has an alphabet of size 8, with $P(X = 1) = 0.35$, $P(X = 2) = 0.175$, $P(X = 3) = 0.175$, $P(X = 4) = 0.08$, $P(X = 5) = 0.08$, and $P(X = 6) = 0.08$, $P(X = 7) = 0.04$, and $P(X = 8) = 0.02$,

a. (5 points) Compute the entropy, $H(X)$, in bits.

b. (15 points) Design a Huffman code for this source. Specify the codeword for each source letter, and compute the average number of codeword bits per source letter, $\bar{\ell}$. Compare the average codeword length to the entropy.

10. (15 points) A baseband (PAM) digital signaling system is to be designed to transmit 4,800 bits/sec over a baseband channel of bandwidth 1,100 Hz (i.e., the frequency band [0, 1100] Hz is available). Assume a raised cosine pulse with spectral rolloff parameter α is to be used, and determine a suitable design for the system by specifying:

i) the number of bits per symbol, R;

ii) the symbol rate, $1/T$;

iii) the value of α for the raised cosine pulse;

iv) the required bandwidth for your design.

11. (20 points) Design a Butterworth filter to satisfy the following frequency response specifications:

i) the gain is no less than -1 dB for $|f| \leq 2,000$ Hz.

ii) the gain is no larger than -25 dB for $|f| > 5,000$ Hz.

For your design, determine N, the required number of poles in the Butterworth filter transfer function, and ω_c, the 3 dB cutoff frequency.

Useful Formulae

Define the "boxcar" time and frequency pulses as

$$p_T(t) = \begin{cases} 1, & \text{for } |t| \leq T; \\ 0, & \text{otherwise.} \end{cases}$$

$$P_W(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq W; \\ 0, & \text{otherwise.} \end{cases}$$

Define the sinc pulse as

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

Various Fourier Transform Relations

$$\mathcal{F}\{p_T(t)\} = 2T\ \text{sinc}(\frac{\omega T}{\pi})$$

$$\mathcal{F}\{\cos(\omega_c t)\} = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$

$$\mathcal{F}\{x(t - t_0)\} = X(\omega)e^{-j\omega t_0}$$

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a + j\omega}$$

$$\mathcal{F}\{x(t)y(t)\} = \frac{1}{2\pi}X(\omega) * Y(\omega)$$

$$\mathcal{F}\{e^{-a|t|}\} = \frac{2a}{a^2 + \omega^2}$$

$$\mathcal{F}\{\frac{W}{\pi}\ \text{sinc}(\frac{Wt}{\pi})\} = P_W(\omega) \qquad \mathcal{F}\{\text{sinc}(\frac{t}{T})\} = P_{\frac{\pi}{T}}(\omega)$$

$$|H(\omega)|^2 = \frac{1}{1 + (\frac{\omega}{\omega_c})^{2N}}$$

$$h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h(n)e^{-j\Omega n}$$

$$f = ma \qquad a = \pi r^2 \qquad e = mc^2$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

Raised Cosine Pulse Bandwidth: $BW = \frac{1+\alpha}{2T}$