TEST 2

EE 341 — Fall 2007

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Closed book, one hour (50 minutes) exam. Two (double-sided) pages of notes are allowed. Some useful formulae, including several Fourier Transform pairs, are appended.

1. (10 points) You are to perform computer simulations of a binary digital communication system and use the simulations to estimate the probability of bit error. If you want to estimate a probability of bit error of about 10^{-4}, transmission of how many bits should be simulated?

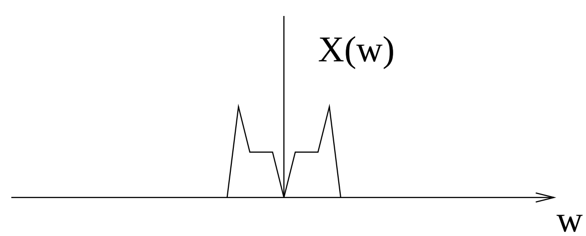
a. at most 10^3 bits.

b. about 10^4 bits.

c. at least about 10^6 bits.

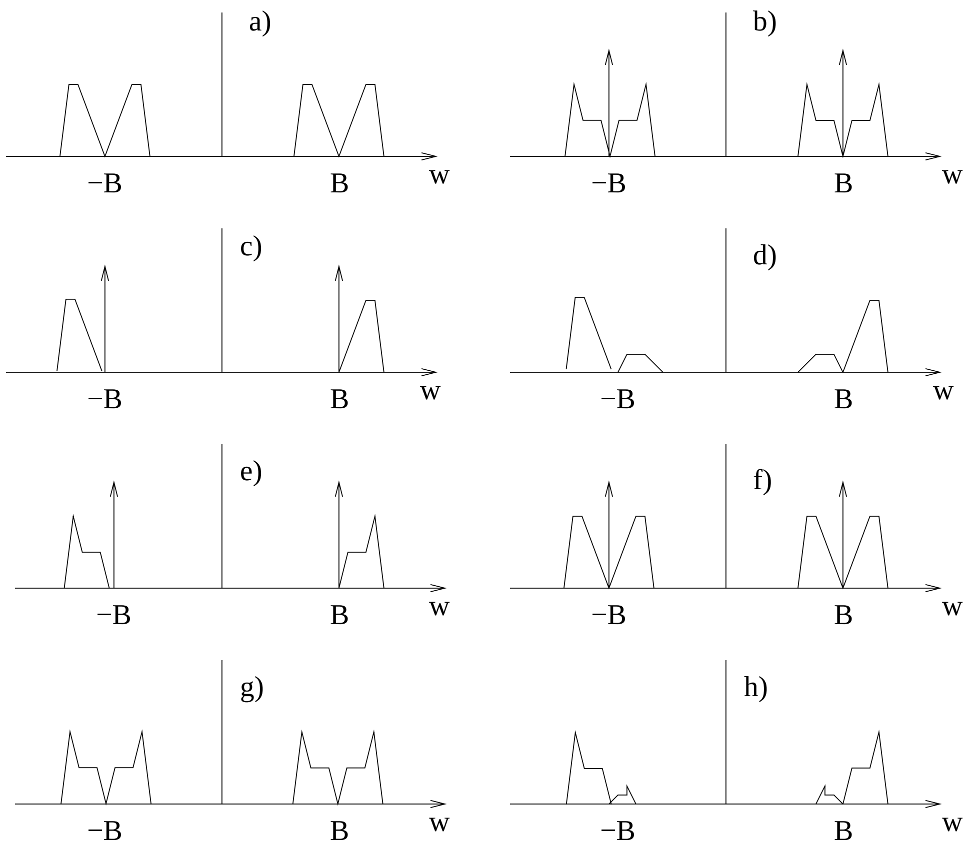
d. no more than 10 bits.

2. An analog signal, $x(t)$, has the Fourier Transform, $X(\omega)$, shown below.



a) (10 points) A signal $y_a(t)$ is formed as $y_a(t) = x(t)\cos(Bt)$, where B is larger than the bandwidth of $x(t)$. Find, in terms of $X(\omega)$, the Fourier Transform of $y_a(t)$, and identify the spectrum of $y_a(t)$ in the figure on the next page.

b) (10 points) A signal $y_b(t)$ is formed as $y_b(t) = (1 + \mu x(t)) \cos(Bt)$, where B is larger than the bandwidth of $x(t)$. Find, in terms of $X(\omega)$, the Fourier Transform of $y_b(t)$, and identify the spectrum of $y_b(t)$ in the figure on the next page.



3. (15 points) An analog signal, $x(t)$, is applied as input to a linear, time-invariant system with impulse response $h(t)$ and frequency response $H(\omega)$ to produce the system output $y(t)$. Find $y(t)$ for input $x(t) = 2\cos(2\pi 100t) - 3\sin(2\pi 250t) - \cos(2\pi 600t)$ and

$$H(\omega) = \begin{cases} 2e^{-j\omega t_0}, & \text{for } 300\pi \leq |\omega| \leq 600\pi; \\ 0, & \text{otherwise;} \end{cases}$$

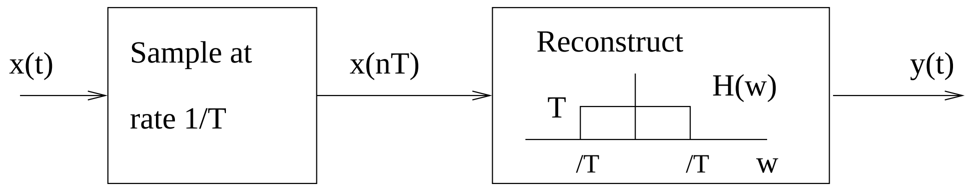
where $t_0 = 0.003$ sec.

4. (20 pts) A Butterworth filter is characterized by two parameters: the 3 dB cutoff frequency, ω_c, and the number of poles (filter order), N. Design a Butterworth filter, with minimum filter order, to satisfy the following specifications:

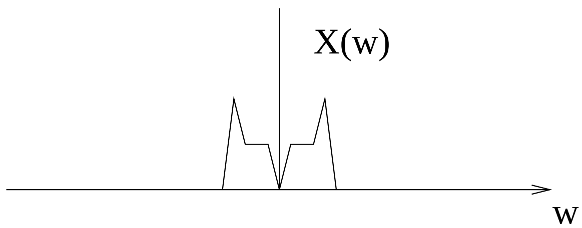
i. The filter gain is no smaller than -1 dB for frequencies less than or equal to 100 Hz.
ii. The filter gain is no larger than -20 dB for frequencies greater than or equal to 500 Hz.

Determine the filter cutoff frequency, ω_c, and minimum filter order, N, to satisfy the specifications.

5. A continuous-time signal $x(t)$ is sampled at rate $1/T$ samples/sec, and the samples are used to construct the waveform $y(t)$, as shown below.



a. (5 points) If the spectrum for $x(t)$ is as shown below, what, according to the sampling theorem, is the minimum sampling rate that can be used to allow exact reconstruction of the signal from its samples?



b. (15 points) Let $x(t) = \sin(2\pi 600t) - 2\cos(2\pi 350t)$. If $1/T = 1000$ Hz, find $y(t)$, the output of the reconstruction filter. First find $X(\omega)$, then the spectrum of the samples, then find $Y(\omega)$, and finally find $y(t)$.

c. (15 points) The signal $x(t) = \text{sinc}(100t)\cos(2\pi 500t)$ is applied as input to the sampler. Note that $\text{sinc}(100t)$ has Fourier transform

$$\frac{1}{100}P_{100\pi}(\omega) = \begin{cases} \frac{1}{100}, & |\omega| \leq 100\pi; \\ 0, & |\omega| > 100\pi. \end{cases}$$

Let the sampling rate be $1/T = 800$ samples/sec and find the reconstructed signal

$$y(t) = \sum_n x(nT)\text{sinc}(\frac{t-nT}{T}).$$

Do this by i) Sketching the signal spectrum at the output of the sampler; ii) Using frequency-domain filtering to find $Y(\omega)$; and iii) then finding $y(t)$ using the inverse Fourier transform.

Useful Formulae

Define the "boxcar" time and frequency pulses as

$$p_T(t) = \begin{cases} 1, & \text{for } |t| \le T; \\ 0, & \text{otherwise.} \end{cases} \qquad P_W(\omega) = \begin{cases} 1, & \text{for } |\omega| \le W; \\ 0, & \text{otherwise.} \end{cases}$$

Define the sinc pulse as

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

Various Fourier Transform Relations

$$\mathcal{F}\{p_T(t)\} = 2T\ \text{sinc}(\frac{\omega T}{\pi})$$

$$\mathcal{F}\{\cos(\omega_c t)\} = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$

$$\mathcal{F}\{\sin(\omega_c t)\} = \frac{\pi}{j}[\delta(\omega - \omega_c) - \delta(\omega + \omega_c)]$$

$$\mathcal{F}\{x(t - t_0)\} = X(\omega)e^{-j\omega t_0}$$

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a + j\omega}$$

$$\mathcal{F}\{x(t)y(t)\} = \frac{1}{2\pi}X(\omega) * Y(\omega)$$

$$\mathcal{F}\{e^{-a|t|}\} = \frac{2a}{a^2 + \omega^2}$$

$$\mathcal{F}\{\frac{W}{\pi}\ \text{sinc}(\frac{Wt}{\pi})\} = P_W(\omega), \quad \text{so} \quad \mathcal{F}\{\text{sinc}(\frac{t}{T}) = TP_{\frac{\pi}{T}}(\omega)$$

$$|H(\omega)|^2 = \frac{1}{1 + (\frac{\omega}{\omega_c})^{2N}}$$

$$\sum_{n=\infty}^{\infty} x(nT)e^{-j\omega nT} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - \frac{2\pi k}{T})$$

Other Formula

$$f = ma \qquad a = \pi r^2 \qquad e = mc^2$$

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)\text{sinc}(\frac{t - nT}{T})$$

$$\frac{\frac{1}{T}\int_0^T |x(t)|^2 dt - \sum_{n=-N}^{N} |c_n|^2}{\frac{1}{T}\int_0^T |x(t)|^2 dt}$$