Methods for Calculating Oscillations in Large Power Systems

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Abstract - We present algorithms to calculate the Hopf bifurcation related segment of the feasibility boundary for a realistically large power system model. The algorithms are used by calculating the Hopf related segment of the feasibility boundary, for the well-recorded June 12, 1992 disturbance on the midwestern segment of the US interconnection system for analyzing the resulting oscillations. The computational results confirm that this event was indeed related to a Hopf bifurcation. This proposed method clarified the operation perspectives of the low frequency power oscillation that occurred. Also it demonstrated the value of the proposed computational approach for assessing the security of an operating point in an area of the parameter space.

1. INTRODUCTION

Transient oscillatory swings are among the common stability problems encountered by the power industry. Typically modal analysis of linearized power system models is carried out together with transient stability studies for identifying the cause of such oscillatory phenomena and suitable corrective control actions are initiated. Slowly damped or sustained oscillations which are associated with certain system compositions and operating conditions while also quite common, are less well understood and relatively much harder to analyze than ordinary transient stability problems. In this paper, we present a methodology for analyzing the occurrence of such oscillatory phenomena in large scale power system models using Hopf bifurcation computations.

The motivation for the development of these algorithms came from a particular oscillatory event which occurred in the Midwestern US as summarized in Section 2 of this paper. The utility involved in the event had previously carried out exceptionally thorough preliminary numerical analysis of the event which pointed to the potential role of a Hopf bifurcation. A Hopf bifurcation occurs when a pair of complex conjugate eigenvalues move from having negative real parts to having positive real parts by crossing the imaginary axis under parameter variations. During this midwestern event, the relay action which led to sustained oscillations moved the system operating condition from being small-signal stable before the event to becoming small-signal unstable in the post-contingency configuration. Specifically, a pair of complex conjugate eigenvalues which had negative real parts (i.e. positive damping) before the event changed to a mode with positive real parts (i.e. negative damping) after the contingency. Therefore, the relay actions moved the system parameters across the Hopf bifurcation related feasibility boundary which resulted in small-signal instability and sustained oscillations.

In this paper, several algorithms are presented for the computation of Hopf bifurcations and these algorithms are tested on the data from an actual 1992 Midwestern disturbance event affecting a large system of 7500 buses and 5700 dynamic states. It is shown that the results presented here are indeed consistent with recordings of the actual event. Moreover these computations, based on a concept of feasibility regions[14,16], provide valuable insight into the operational perspectives of the oscillatory event. Existing large scale, commercial software packages including EPRI’s PSAPAC programs and PTI’s PSS/E programs, which perform computation routines for getting conventional results (like power-flow, eigenvalues, equilibrium points, etc.) are embedded in the overall program developed by the authors. This reduces the programming effort in implementing the algorithms on large scale systems.

The analysis based computational results confirm that the Rush Island event mentioned above, was indeed related to a Hopf bifurcation. A piece of the Hopf related segment of the
feasibility boundary [14,16] for the event was computed to illustrate its usefulness in judging security in the feasibility region in practical systems.

The concepts and the mathematical preliminaries are introduced in Section 3. The Hopf bifurcation computational algorithms are presented in Sections 4 and 5. Simulations of the actual event data follow in Section 6.

2. SUMMARY OF AN OSCILLATORY DISTURBANCE ON THE US INTERCONNECTION ON JUNE 12, 1992

On June 12, 1992, at 12:46 pm, a standoff insulator failed at UE's Tyson 345 KV substation [12]. This was followed by a sequence of relaying and switching actions which left an unusual system configuration as shown in Fig. 1. In this post-fault condition, Rush Island units were connected to the transmission system through only one outlet circuit with two major north-south transmission ties severed. Generation at Rush Island was at about 1100 MW at the time of the fault.

After the condition shown in Fig. 1 set in, UE and neighboring generating units experienced an oscillation in the magnitude of power output and of voltage for about 38 minutes. At Rush Island, the total plant output was oscillating about 280 MW, swinging between a peak value of about 1200 MW and a minimum of about 920 MW. The frequency of the oscillation at the Rush Island plant was observed to be about one Hertz. Other plants on the UE system and surrounding systems experienced MW oscillations of a lower magnitude in the range of 25 to 75 MW.

The only major adjustment in-progress at a UE plant when the oscillation ceased at about 13:24 was the lowering of the MW output level at Rush Island (see Fig. 2). Reducing Rush Island generation has been verified to be the principal cause of oscillation cessation.

This event is well documented by simulation and other studies as well as the original recordings of the event [12]. A comprehensive analysis conducted by the UE engineers showed the operating equilibrium point was small-signal unstable for the post-contingency configuration and moreover their time domain simulation using ETMSP illustrated the existence of a stable limit cycle. This combination indicates the existence of a possible supercritical Hopf-bifurcation. Here we analyze the event; establish its characterization; and compare the conclusions with the known physical facts. The algorithms introduced will in general be useful for predicting small-signal stability and security near Hopf-bifurcation related segments of the feasibility boundary for any general large such power system model. The computational algorithms should prove to be useful in general.

Our results thus demonstrate the applicability of Hopf-bifurcation related methods proposed in [7,17] on real systems of large size.

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Fig. 1. Postfault system configuration.

Fig. 2. Chart record of the Rush Island Event on June 12, 1992.
3. THE MATHEMATICAL MODEL

The smooth quasi-stationary dynamics of the large electric power system can be modelled by a parameter dependent differential-algebraic equation (DAE) system \( \Sigma \) of the form as established in [14].

\[
\begin{align*}
\dot{x} &= f(x, y, p), \quad f : \mathbb{R}^{n+m+r} \to \mathbb{R}^n \quad (1) \\
\Sigma : \quad 0 &= g(x, y, p), \quad g : \mathbb{R}^{n+m+r} \to \mathbb{R}^m \quad (2) \\
x &\in X \subset \mathbb{R}^n, \quad y \in Y \subset \mathbb{R}^m, \quad p \in P \subset \mathbb{R}^r \quad (3)
\end{align*}
\]

In the state space \( X \times Y \), dynamic state variables \( x \) and instantaneous state variables \( y \) are distinguished. While the (dynamic) variables \( x \) have their associated dynamics explicitly modelled by (1), the dynamics of the (instantaneous) variables \( y \) is assumed to be so fast that the constraints \( g = 0 \) in (2) are always satisfied. The parameters \( p \) define a specific system configuration and the operating condition.

For the power system, typical dynamic state variables are the time dependent values of generator voltages and rotor angles, control states, and load dynamics. and instantaneous variables are the transmission line power-flow variables (the bus voltages and angles). The parameter space \( P \) is composed of system parameters (which describe the system topography, i.e. which lines, buses etc. are energized, and equipment constants such as inductances, power ratios etc.), and operating parameters (such as loads, generation, voltage setpoints, etc.). The dynamics of the generators, control devices and the load dynamics together define the \( f \) equations. Typically the constraints \( g = 0 \) are defined by the power balance equations of the transmission system [14].

A mathematical analysis of the DAE system \( \Sigma \) shows that the operating equilibrium point remains stable, i.e. valid within a region which we call the feasibility region [15]. Given a stable equilibrium point \((x_0, y_0)\) for parameter value \( p_0 \), the feasibility region of \((x_p, y_p, p_0)\) is defined as the set of all stable equilibrium points, which can be reached quasi-statically (or in a stationary sense) from \((x_p, y_p, p_0)\) by continuous parameter variations without losing local stability. The boundary of the feasibility region is called the feasibility boundary. Note that the feasibility region is defined as a subset of \( X \times Y \times P \), not a subset of the parameter space. However, typically only one solution will satisfy the viability requirements hence qualify as an actual operating point. Therefore typically the viable portion of the feasibility region has a well-defined projection into the parameter space [14]. This projection of the feasibility region and its boundary are used exclusively in this paper.

The loss of local stability at the operating equilibrium point is one of the defining features of local bifurcation points [5]. Boundary segments of the feasibility region so correlated for the differential algebraic equation \( x = f \) and \( g = 0 \), typically, will be contained in one of three different local bifurcation sets which are 1) saddle-node bifurcations (connected with zero eigenvalues); 2) Hopf bifurcations (connected with purely imaginary complex conjugate eigenvalues); and 3) singularity induced bifurcations (connected with singularity of the network equations and, unbounded eigenvalues) [16]. This paper deals with the computation of the Hopf bifurcation related feasibility boundary and the verification of the Rush Island event as a Hopf related oscillation.

One must remember that the word feasibility is widely used throughout engineering. The basic sense is the same but for individual applications the precise meaning has to be defined. The above mentioned definition is useful for studying small-signal stability properties of system operation and security in a diverse environment which is one of the objectives in the Taxonomy theory [14].

4. METHOD FOR COMPUTING A HOPF BIFURCATION POINT

The significance of Hopf bifurcation in power system models was first identified in [1]. Hopf bifurcations have been studied in many recent papers including [3,10,14]. Computational methods for calculating nearest Hopf bifurcations have been studied in [3]. More general algorithms for computing critical eigenvalues of a power system model have been proposed in [8,9,13] for studying small-signal stability problems.

In preparation for computing the Hopf bifurcation related segment of the feasibility boundary, we first consider an effective method for computing the Hopf bifurcation point. We propose an indirect method similar to Hassard's method [6].

Let \( \mu \) be a specific but arbitrarily selected parameter such as a generator output or a gain in the control of a generator, (preferably a parameter to which the bifurcation is sensitive). Let \( \mu^H \) be a parameter value \( \mu \) at which a Hopf bifurcation occurs in the system. Assume that \( \mu \) is sufficiently close to the Hopf bifurcation point \( \mu^H \). The full Jacobian matrix

\[
\begin{pmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{pmatrix}
\]

of the functions \( f \) and \( g \) evaluated at \( \mu = \mu^H + \epsilon \) is then nonsingular for sufficiently small \( \epsilon \). (Here \( f_x, f_y, g_x, \) and \( g_y \) denote the matrices of partial derivatives of \( f \) and \( g \) with respect to \( x \) and \( y \).) Thus, by the implicit function theorem, there exists a smooth curve of equilibrium points, i.e., stationary solutions \( (x', y') \) of (1) and (2) which can be solved in terms of \( \mu \) [14]. And the equilibrium point \( (x', y') \) depends continuously on \( \mu \). So do the elements of the system matrix \( \Lambda = f_x - f_y g_y^{-1} g_x \) evaluated at \( (x', y') \). Hence, the eigenvalues
\( \lambda \) of \( A \) also depend continuously on \( \mu \). Furthermore, there is a pair of purely imaginary complex eigenvalues (Hopf bifurcation eigenvalues) for the Jacobian matrix \( A \) evaluated at \( \mu^h \).

It follows that for some value \( \mu = \mu^h + \varepsilon \), sufficiently small \( \varepsilon > 0 \), there will be a pair of complex eigenvalues which becomes Hopf eigenvalues as \( \mu \) approach to \( \mu^h \). These eigenvalues will be called the critical eigenvalues. This implies that the Hopf bifurcation point can be determined by testing the real parts of the eigenvalues \( \lambda \) of the system matrix \( A \). This method requires repeated evaluation of the eigenvalues for a sequence of starting points generated by some continuation method. We shall assume that among the eigenvalues in some neighborhood of \( \mu^h \) only the Hopf eigenvalues have zero real part. Thus, the Hopf bifurcation point can be characterized by the following set of conditions:

\[
\begin{align*}
f(x, y, \mu) &= 0 \quad (4) \\
g(x, y, \mu) &= 0 \quad (5) \\
h(x, y, \mu) &= \text{Re}(\lambda(\mu)) = 0 \quad (6)
\end{align*}
\]

where \( h(x, y, \mu) = 0 \) denotes a suitably chosen equation which characterizes the purely imaginary eigenvalues of the system matrix \( A \).

When the system matrix \( A \) is transformed using a bilinear transformation [9,13] to \( \tilde{A} = (A + wI) (A - wI)^{-1} \), where \( w \) is a weighting factor assumed to be a real number (which determines the eigenvalue distribution in the z-plane), the Hopf eigenvalues lie on the unit circle and all other eigenvalues are inside the unit circle. That is, the Hopf eigenvalues become the dominant eigenvalues (the eigenvalue with the largest magnitude), which can be easily computed using the power method. The power method (for example, in [4]) is best suited for computing dominant eigenvalues of a large matrix where non-power methods would not be practical. Therefore, the bilinear transformation allows us to effectively use the power method for computing the poorly damped mode of a system under the assumption that all the other eigenvalues are well-damped at the operating point. It follows that for some value \( \mu = \mu^h + \varepsilon \), with sufficiently small \( \varepsilon > 0 \), the pair of critical eigenvalues (which will probably become the Hopf eigenvalue) also become the dominant eigenvalues in the z-plane. Then an algorithm can be designed:

1) Using the power method, find a pair of critical eigenvalues at parameter \( \mu = \mu^h + \varepsilon \) sufficiently small \( \varepsilon > 0 \) (after the Hopf bifurcation).

2) Use a method of successive approximations to guide \( \mu \) towards \( \mu^h \) and thus identify a Hopf bifurcation.

This approach greatly simplifies the computational burden for selecting the critical eigenvalues for large power system.

For comparison, in order to find the critical eigenvalue in the Hassard method [6], all the eigenvalues of the Jacobian matrix are calculated in order to select the critical eigenvalues.

More specifically, the magnitude of the Hopf eigenvalues is 1.0 in the z-plane. Thus the Hopf bifurcation point can be located by solving (4), (5), and

\[
\tilde{h}(\mu) = 1 - r_\mu(\tilde{A}) = 0 \quad (7)
\]

where \( r_\mu(\tilde{A}) \) is the spectral radius, i.e. the largest absolute value of all eigenvalues of \( \tilde{A} \). Based on the sufficient conditions (7) together with (4) and (5), we describe an iterative technique for finding a Hopf bifurcation point \((x^h, y^h, \mu^h)\) on the feasibility boundary.

Let us assume that there exists only one pair of critical eigenvalues (poorly damped or unstable eigenvalues which will probably become the Hopf eigenvalue). Then a pair of critical eigenvalues can be obtained using the power method [4] in the z-plane i.e. power iteration with bilinear transformation \( \tilde{A} = (A + wI_n) (A - wI_n)^{-1} \) with weighting factor \( w=8 \) (see [17] for details on the choice of \( w \)). We select the most sensitive parameter \( \mu \) to these critical eigenvalues (see [7] for detailed method). Then a point \( \mu^h \) on the feasibility boundary can be found by an iterative sequence. The detailed procedure for computing a Hopf bifurcation point is summarized from [17] as follows:

We denote the equilibrium corresponding to the parameter value \( \mu_m \) in the m-th step by \((x_m^e, y_m^e)\) and \( \lambda_m^e \) denotes the critical eigenvalue of the system matrix \( A = f_x - f_y g_y^{-1} g_x \) evaluated at \((x_m^e, y_m^e)\). If \( u = (u_i) \) and \( v = (v_i) \) denote the corresponding left- and right-eigenvector, normalized so that \( u^Tv = 1 \), then the participating factor \( p_{ic} \) of the i-th component is defined by \( p_{ic} = u_i v_i \) [17].

Step 1: Compute the equilibrium \((x_m^e, y_m^e) = (x^e (\mu_m), y^e (\mu_m))\) satisfying \( f = 0 \) and \( g = 0 \) for \( \mu_m \). This process requires an iterative process like Newton-Raphson method. To handle the large size system as well as various types of dynamic devices, we directly use EPRI's PSAPAC programs and power-flow programs.

Step 2: Evaluate the coefficient matrices, \( f_x, f_y, g_x, g_y \) at \((x_m^e, y_m^e)\). For this process, we directly use EPRI's PSAPAC programs.

Step 3: If a priori information on the critical eigenvalue \( \lambda_m^{e-1} \) is available, compute the critical eigenvalue using the inverse power iteration [4], i.e. by power iteration of \((A - \lambda_m^{e-1} I_n)^{-1}\). If the critical eigenvalue is not available, do the bilinear transformation with weighting
factor $w=8$ i.e., $\tilde{\lambda} = (A + wI_p) (A - wI_p)^{-1}$ and compute the critical eigenvalue using the power method or the modified Arnoldi method (see [17] for details). 

The eigenvalue obtained is tested using the convergence criterion $\left| \lambda_{m}^{k} - \lambda_{m-1}^{k} \right| / \left| \lambda_{m-1}^{k} \right| \leq \varepsilon_{\lambda}$.

Step 4: Compute the participation factors $p_{m}$ [17] and verify whether or not it has the same state mode as in the previous iteration step. If so, go to Step 5; otherwise go to Step 6 after replacing a weighting factor $K$ with $0.5K$ and $\tilde{h}_{m} = \tilde{h} (\mu_{m})$ with $\tilde{h}_{m-1} = \tilde{h} (\mu_{m-1})$.

Step 5: Compute the quantity $\tilde{h}_{m} = \tilde{h} (\mu_{m})$ representing the distance of this critical eigenvalue from the unit circle.

Step 6: Test the stopping condition for $\mu$. If either $|\tilde{h} (\mu_{m})| \leq \varepsilon_{h}$ or $|\mu_{m} - \mu_{m-1}| / |\mu_{m-1}| \leq \varepsilon_{\mu}$ is satisfied, the secant iteration stops. If not, go back to Step 1 after determining a parameter $\mu_{m-1} = \mu_{m-1} - \tilde{h}_{m} (\mu_{m} - \mu_{m-1}) / \left[ \tilde{h}_{m} - \tilde{h}_{m-1} \right]$.

5. Method for Computing the Hopf Bifurcation Related Feasibility Boundary

In Section 4, we introduced the iterative method for finding a Hopf bifurcation point. This method is used for a one dimensional parameter vector $\mu$. For a two dimensional parameter subspace, a Hopf related segment is a connected curve consisting of points in the parameter subspace $(\mu', \mu^2)$, that are solutions of

$$\tilde{h} (\mu', \mu^2) = 0,$$

where $\mu', \mu^2 \in \mathbb{R}^2$. Using the algorithm described in Section 4, one solution of (8) has been determined. Let us denote this first solution by $\mu_1 = (\mu_1', \mu_1^2)$.

For further solutions, $\mu_2, \mu_3, ..., \mu_N$, we use predictor-corrector methods (for examples, in [2,11]). Then the Hopf bifurcation segment can be determined by the set of these points, i.e., by simply connecting these solutions.

The predictor provides an approximation of the $k$-th solution $\tilde{\mu}_{k+1}$ of (8) using the solutions obtained in the previous steps. The $(k+1)$-th solution $\mu_{k+1}$ can be found by the corrector iterations with a predicted point $\tilde{\mu}_{k+1}$.

Details of the predictor-corrector method used in this paper are described by the following four parts:

1) Predictor: An approximate solution $\tilde{\mu}_{k+1}$ of (8) is obtained using the secant prediction which can be expressed as

$$\tilde{\mu}_{k+1} = \mu_k + \beta_k (\mu_k - \mu_{k-1}) = \mu_k + \Delta_{k} \mu_{i}$$

where the quantity $\beta_k$ is a weighting factor.

2) Parametrization: In this paper, the local parametrization is used. That is, one of the variables $\mu_k = (\mu_{k1}, \mu_{k2})$ is used as parameter. This leads to the parametrizing equation

$$\mu_{i}^k = \eta = 0$$

with an index $i$ and a suitable value of step length $\eta$. The index $i$ and the parameter $\eta$ are locally determined at each continuation step in order to keep the continuation flexible. Hence, we have the augmented equations

$$
\begin{pmatrix}
\tilde{h} (\mu_{k1}, \mu_{k2}) \\
\mu_{i} - \eta
\end{pmatrix} = 0 \quad i = 1 \text{ or } 2.
$$

Based on the secant predictor (9), $i$ is determined such that the relative changes $\Delta_{k}$ are maximal in $\mu_{k}$,

$$\Delta_{k} = \max (\Delta_{k1}, \eta, \Delta_{k2})$$

where $\gamma$ is a scale factor for the normalization of the $i$-th parameter.

3) Step length: We use an adaptive step length $\eta = \Delta_{k}^{max}$, where $\Delta_{k}^{max}$ is the maximum variation of the $i$-th parameter. $\Delta_{k}^{max}$ is adjusted during the iteration. If the predictor iteration failed, the previous iteration is repeated with $\eta = 0.5 \Delta_{k}^{max}$. Once a local parameter by (12) has been selected, say the $i$-th parameter, the $k$-th step length is determined by

$$\beta_k = \eta / \Delta_k^i.$$

Note that $\Delta_k^{max}$ and $\eta$ for $i = 1, 2$ should be specified. Basically the $k$-th parameter is stretched to the maximum length specified and other parameter is stretched proportionally.

4) Corrector: The $k$-th continuation step starts from an approximation $\tilde{\mu}_{k+1}$, of a solution $\mu_{k+1}$ of (8). In general, the $\tilde{\mu}_{k+1}$ is not a solution of (8). The predictor merely provides an initial guess for corrector iterations which home in on a solution of (8). The augmented equation (11) is solved with a suitable index $i$ and value of $\eta$ by the proposed method for computing a Hopf bifurcation point described in Section 4.

![Fig 3 Basic scheme of predictor-corrector method](image-url)
6. COMPUTATIONAL ANALYSIS

A complete algorithmic computational process was developed and programmed by incorporating the use of large commercial programs (EPRI's PSAPAC programs and PTT's PSS/E programs). All this was summarized in Section 3-5. Computational illustrations follow.

A. General Considerations

Models of all types of equipment in the power systems, except the composite loads as seen at the transmission buses, are well understood and very highly detailed physically based dynamic models for equipment like generators, transformers etc. are presented and programmed in EPRI's software packages (SSSP; ETMSP, etc.). System loads are modelled by constant impedance representation. The results in the paper do not change appreciably when other types of load models are considered as shown in the detailed report [17].

B. System Modeling

We consider a regional system which represents the midwestern U.S. interconnected system. Rush Island is in this regional system. Thus by using the regional system model the system is modeled in its entirety. This is a large system of 7600 buses and 5700 dynamic states, and realistic system models are used. Knowledge of detailed system operating conditions in and around the UE area at the termination of the system are also assumed as in [17].

It is of course not feasible to present all the data of this model (7600 buses and 5700 dynamic states) and the record of its loading at the time of the event. However, some of these data are available in our final report to EPRI [17].

This is then an illustration of the feasibility boundary theory on systems of real size.

C. Selection of System Parameters for Computational Analysis

While an oscillation is present on the system, the first action taken by the operator may be changing the status, controls, and/or settings of both the excitation and steam supply systems in an effort to eliminate the oscillation. This is what happened at Rush Island.

Among the most sensitive parameters to critical eigenvalues, we selected three parameters, real power generation \( P_g \), forward gain \( K_a \), and feedback gain \( K_f \) of the excitation system automatic voltage regulator (AVR) for computing the Hopf bifurcation segments (see [7] for details on the choice of these parameters). There are two generator units at Rush Island. Both units have the same governor and excitation systems. However, to make the display of results more effective, we use a variation of three parameters \( (P_g, K_f, K_f) \) for only Rush Island Unit 1 will be considered as follows:

Case #1: Real power generation \( P_g \) at Rush Island Unit 1 and forward gain \( K_a \) of AVR of Rush Island Units 1.

Case #2: Forward gain \( K_a \) and feedback gain \( K_f \) of AVR of Rush Island Unit 1.

Note that Fig. 2 shows that major activity was concentrated in Unit 1 among the two generators during the time of the actual event.

D. Computation of the Hopf Related Segment

The system operating conditions used for all computational results following are based on calculated design values and post-fault operating conditions for the June 12, 1992 event. The data for the figures below are given in [17]. Here we describe the main features only.

In order to get an initial Hopf point on the feasibility boundary, the method described in Section 4 is used. For computing the Hopf related segment of the feasibility boundary, a continuation algorithm described in Section 5 is used.

1) Case #1, \( (P_g, K_a) \): The Hopf related segment of the feasibility boundary is computed in the two dimensional parameter subspace \( (P_g, K_a) \), where \( P_g \) and \( K_a \) are real power generation and the forward gain of the excitation system AVR for Rush Island Unit 1, respectively. Generation at Rush Island was at about 1100 MW at the time of the fault. (543 MW for Unit 1 and 551 MW for Unit 2.) The calculated value (design value) of the forward gain \( K_a \) of AVR in the excitation system for Rush Island Unit 1 is 750. To get the Hopf related segment, the ranges of these two parameters must be defined. For convenience, we select a practical range for each parameter as 200 \( \leq P_g \leq 600 \) and 200 \( \leq K_a \leq 900 \).

All other parameters for Unit 1 such as \( K_f, T_f \), etc., are fixed with the design operating values. All parameters for Unit 2 are fixed as well [17].

First we locate the Hopf bifurcation point \( P_h \) of the given system for \( K_a = 900 \) fixed using the proposed method described in Section 4. This point is used as the starting point for the continuation method proposed in Section 5. Results obtained by this procedure for the regional system are illustrated in Fig. 4. Note that, as shown in Fig. 4, post-fault operating point of Unit 1 is in the infeasible region. Within the infeasible regions of the parameter space, the equilibrium point of the system is small-signal unstable.

Time domain simulations are performed at two operating
points, one with parameter inside the feasible region and the other with parameter outside the feasible region. EPRI's Extended Transient/Midterm Stability Package (ETMSP) was used. Fig. 5 shows the MW outputs of the Rush Island Unit 1 from the time domain simulation. When the governor reference is changed from 1 p.u. to 0.8 p.u. at time 10 seconds as a step change of the parameter (the parameter is outside the feasibility boundary), the time domain simulations showed a stable limit cycle (Fig. 5.a). This limit cycle disappeared as the parameter crossed the feasibility boundary into the feasible region, i.e., the governor reference is changed from 1 p.u. to 0.6 p.u. at time 10 seconds (Fig. 5.b). The corresponding phase portraits for each case are also shown in Figs. 6.a and 6.b, respectively.

In the Rush Island event the real power generation of Unit 2 was reduced slowly compared to Unit 1 apparently following a schedule, then in about the last 10 minutes it was reduced much faster. The total change during the oscillation was about 200 MW for Unit 1 and about 80 MW for Unit 2. To explore this type of event the feasibility boundary was computed with Unit 1 output variable and Unit 2 output fixed at the post disturbance value at 80 MW's below the value before the disturbance. The results are shown for comparison in Fig. 7.

The post fault system conditions and the system conditions at the termination of the system oscillation are indicated with a cross (‘x’) respectively in the right side and the left side of Fig. 7. From Fig. 7 the sustained oscillation of the Rush Island event following the system disturbance can be explained. The system resided in the infeasible region after fault clearance at Ka = 730 and Pg = 543 MW. The system trajectory therefore approaches a stable limit cycle present after the supercritical Hopf bifurcation. As the power generation was reduced the oscillation stopped at about Pg = 350 MW after the state of the systems regained the stable equilibrium point.
2) Case #2, \((K_f, K_a)\): The Hopf related segment of the feasibility boundary is computed in the two dimensional parameter subspace \((K_f, K_a)\), where \(K_f\) and \(K_a\) are the feedback gain and the forward gain of the excitation system AVR for Rush Island Unit 1, respectively.

Based on the actual control setting data, excitation system model parameters were calculated by the excitation system manufacturer [12]. The calculated values (design value) of \(K_f\) and \(K_a\) were 0.01 and 730 respectively for Unit 1.

For convenience, we select a practical range for each parameter as \(0.005 \leq K_f \leq 0.05\) and \(200 \leq K_a \leq 900\), are assumed. All other parameters for Unit 1 such as \(P_g\), \(T_a\), etc. are fixed with the design operating values. All parameters for Unit 2 are fixed as well [17].

Results obtained by this procedure for the regional system are illustrated in Fig. 8. Again, a constant impedance load model was assumed for the results shown in Fig. 8. (see [17] for the effect of different types of static load models). The design data for Unit 1 at post fault system conditions (actual values during the event) are indicated with a cross ('x') in Fig. 8.

Time domain simulations at each parameter point, one in the feasible region, and the other in the infeasible region, were also performed using EPRI’s software ETMSP and the results are shown in Fig. 9. The parameter values for Unit 1 are described in each figure. The parameter values for Unit 2 are fixed at post-fault conditions. The corresponding phase portraits are given in Figs. 10.a and 10.b, respectively. Here we interpret this oscillation in the two dimensional parameter subspace \((K_f, K_a)\). Fig. 8 shows the Hopf segment including the design gain setting indicated with a cross. From Fig. 8, it is clear that the AVR gains were set outside the feasible region at the time of the Rush Island event (post-fault condition). Also it is clear that this oscillation could be prevented if the exciter gains were set to the values inside the feasible region in this two dimensional parameter subspace \((K_f, K_a)\). This fact indicates a valuable application of the feasibility region context for determining a gain setting of the exciter system. In fact, UE planning engineers have successfully used the feasibility boundary computational methods described in this paper together with other tools such as transient stability methods for selection and tuning of exciter control gains in the UE system.

7. CONCLUSION

The computational results confirm that the Rush Island event was indeed related to a Hopf bifurcation. A segment of the feasibility boundary was computed, its applications for evaluating the security of the operating point discussed and its implications and usefulness were discussed and illustrated in Section 6.
This work verifies the practicality of the Hopf bifurcation theory as the basis for precise analysis of oscillatory phenomena in even very large and complex systems modeled in detail. The verification is accomplished here by a study through developing an experimental software that incorporates major EPRI software packages. Of course, there is no novelty claimed for the basic Hopf bifurcation analysis which has been long established historically and also been extensively studied in the literature recently. However, this paper for the first time reports successful development and implementation of computational algorithms for studying Hopf bifurcation related phenomena in large scale power system models using the feasibility boundary concept introduced in [14,15,16]. The techniques have also been verified on an actual disturbance event in a real very large power system.

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9. REFERENCES


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