Optimal Partitioning of Multicast Receivers

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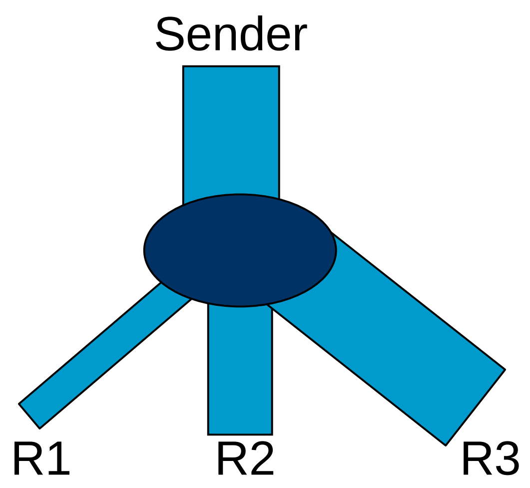
Outline of Presentation

- Motivation
- Optimal Receiver Partitioning
- Max-min Fair Rate Optimal Partitioning
- Experimental Evaluations
- Conclusion

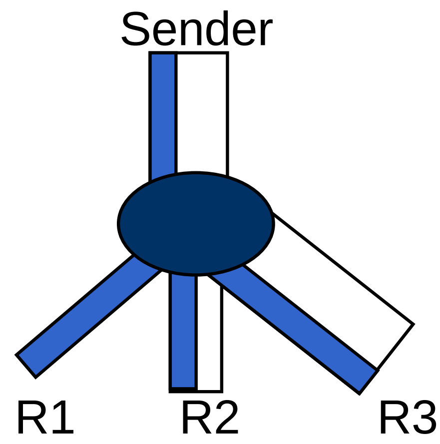
Why Optimal Partition?

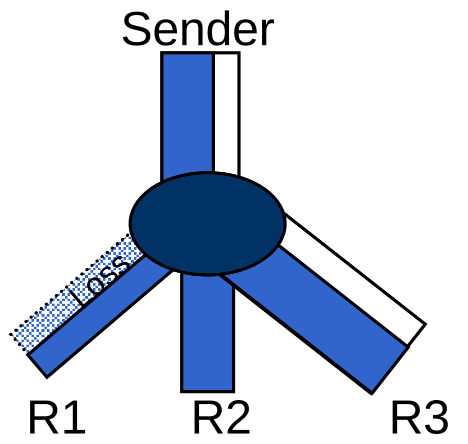
- Heterogeneous receiver capacities
  - Receiver host restrictions
  - Network path: Modem, ISDN, Cable Modem, LAN



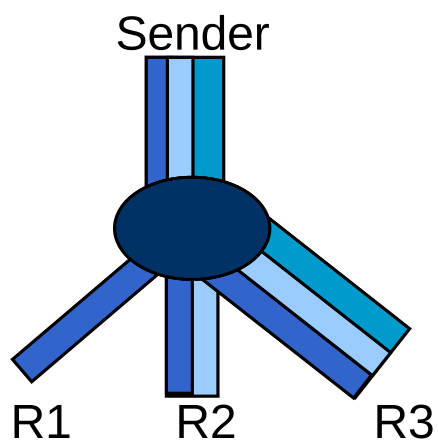
How to determine the sending rate(s)?

- Single-rate
  - The lowest rate
  - To maximize inter-receiver fairness





- Multi-rate
  - Q1: How many groups?
  - Q2: How to determine the rates?



Q1: How many groups?

- As many as receivers
- A fixed number of groups, e.g., 4 groups
- The more groups, the higher
  - the sender overhead to encode
  - the network overhead to keep the states
  - the receiver overhead to decode
- Our result: 4-5 groups for majority of benefits

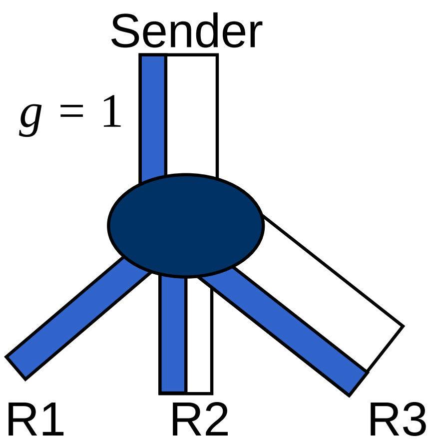
Q2: How to determine the rates?

- Static
  - Independent of receiver capacities
  - Determined by encoding scheme
- Dynamic
  1. Heuristics
  2. Our solution: dynamic programming to find an optimal solution

Answer to Q2: Optimal Receiver Partition

- Terminology
  - Isolated rate r_i
  - Receiver utility function $u(r, g)$
  - Group utility

$$U(G, g) = \sum_{i \in G} u(r_i, g)$$

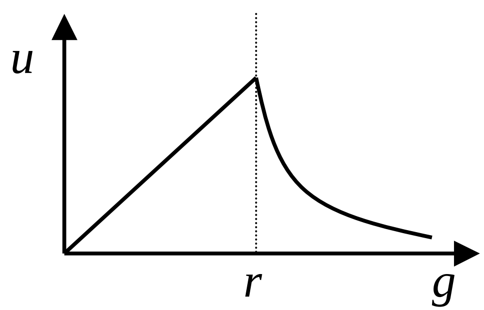


r_i	1	2	3
$u(r_i, g)$	1.0	0.5	0.3

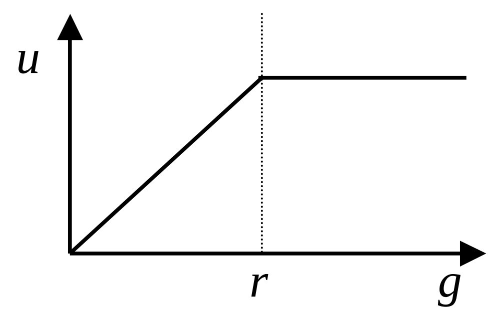
$U(G, g) = 1.0 + 0.5 + 0.3 = 1.8$

Receiver Utility Function

- $u(r, g)$
- Properties
  - Non-decreasing when r and g approach to each other.
  - Maximum when $r = g$.
- Examples



$u(r, g) = \min(r, g) / \max(r, g)$

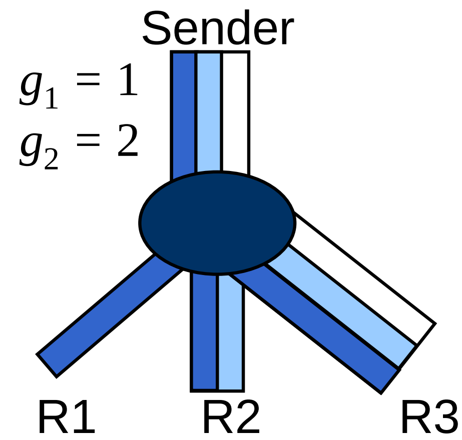


$u(r, g) = \min(r, g)$

Optimal Partition

- Session utility

$$V(\{(G_1, g_1), \dots, (G_K, g_K)\}) = \sum_{k=1}^{K} U(G_k, g_k)$$



r_i	1	2	3
$u(r_i, g_1)$	1.0		
$u(r_i, g_2)$		1.0	0.7

$U(G_1, g_1) = 1.0$

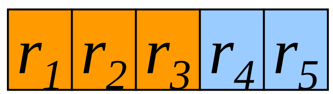
$U(G_2, g_2) = 1.0 + 0.7 = 1.7$

$V = 1.0 + 1.7 = 2.7$

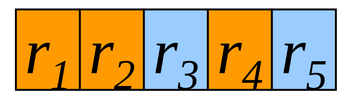
- Optimal partition
  - Maximizes the session utility.

Finding the Optimal Partition

- Ordered partition
  - If $i \in G_k$ and $j \in G_{k+1}$, then $r_i \leq r_j$.

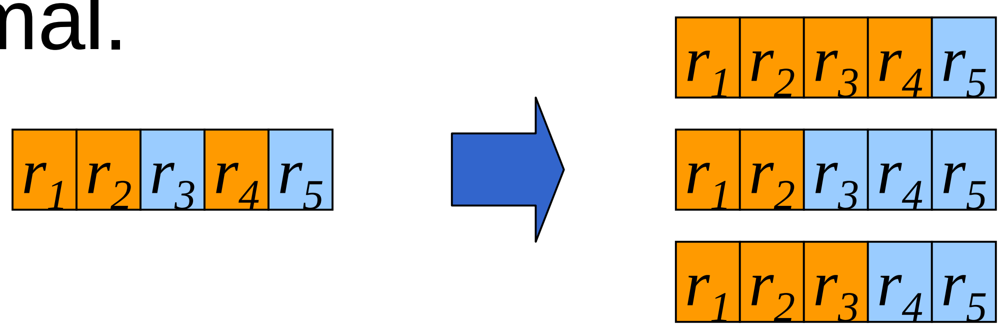


Ordered partition



Unordered partition

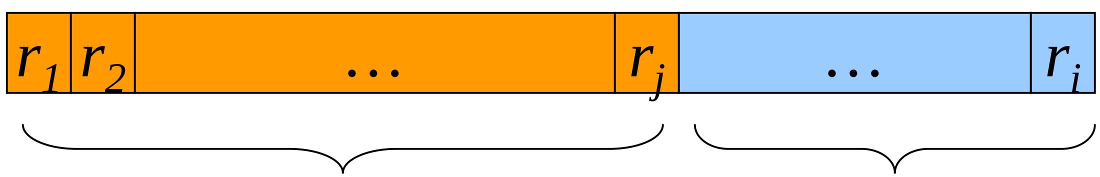
- There exists an ordered partition that is optimal.



Finding the Optimal Partition

- Dynamic programming algorithm
  - Finds the ordered optimal partition

$$V^*(i,m) = \max_{1 \le j < i} \left(V^*(j, m-1) + U^*(\{j+1, \text{😐}, i\}) \right)$$

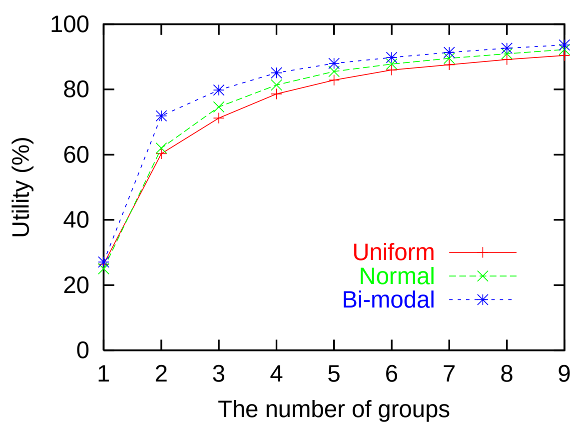


m-1 groups — $V^*(j, m-1)$

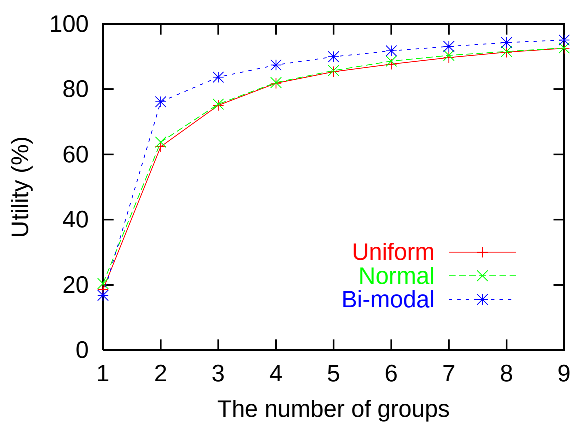
m-th group — $U^*(\{j+1, \text{😐}, i\})$

Answer to Q1: Number of Groups

$u(r, g) = \min(r, g) / \max(r, g)$



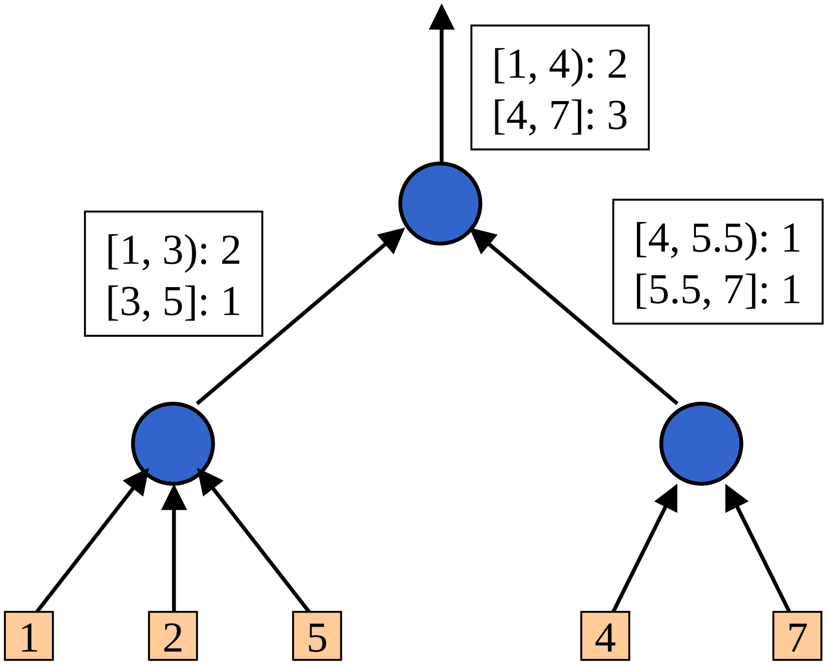
$u(r, g) = \min(r, g)$



- 4 groups for 80% of the maximum.

Collecting Isolated Rates

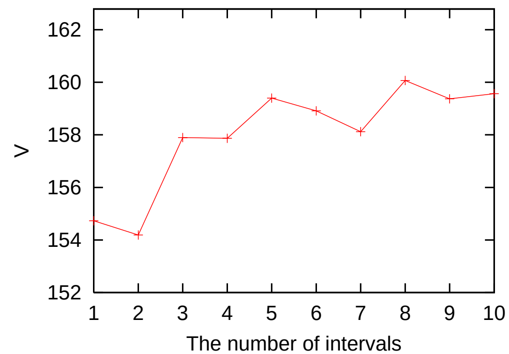
- Max-min fair rates as isolated rates
- Aggregation

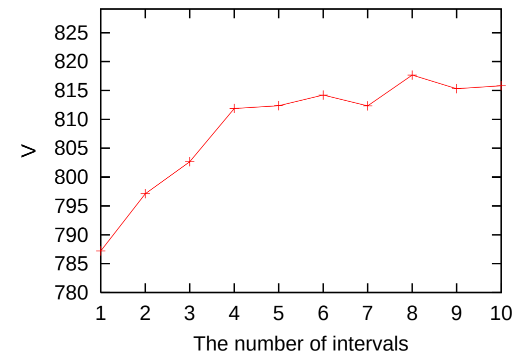


Experimental Evaluations

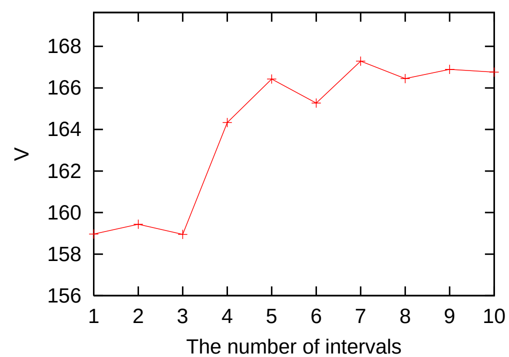
$u(r, g) = \min(r, g) / \max(r, g)$ | $u(r, g) = \min(r, g)$

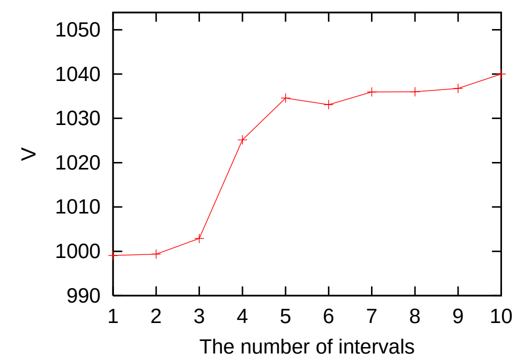
Normal





Bi-modal





- Aggregation error < 3% with 4 intervals.

Conclusion

- Heterogeneous receiver capacities.
- Determine the optimal receiver partition in multi-rate multicasts.
- Achieve 80% of the maximal utility.
- Future work
  - Extension to other network fairness.

The End

Question?