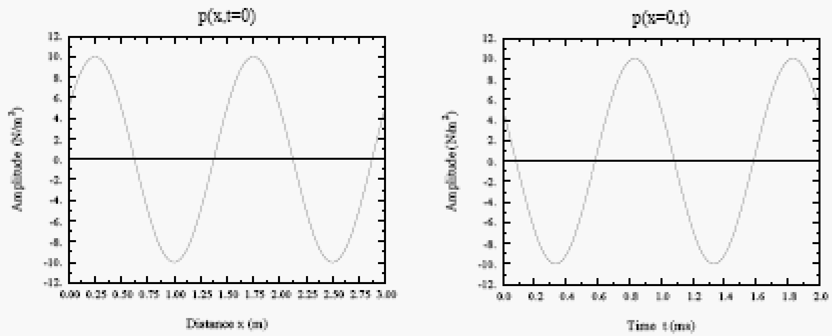
EE331 Homework # 1 Solution

Problem 1.2 For the pressure wave described in Example 1-1, plot
(a) $p(x,t)$ versus x at $t = 0$,
(b) $p(x,t)$ versus t at $x = 0$.
Be sure to use appropriate scales for x and t so that each of your plots covers at least two cycles.

Solution: Refer to Fig. P1.2(a) and Fig. P1.2(b).



(a) (b)

Figure P1.2: (a) Pressure wave as a function of distance at $t = 0$ and (b) pressure wave as a function of time at $x = 0$.

Problem 1.8 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{max} = 40$ cm, $\lambda = 30$ cm, $f = 10$ Hz, and
(a) $y(x,0) = 0$ at $x = 0$,
(b) $y(x,0) = 0$ at $x = 7.5$ cm.

Solution: For a wave traveling in the negative x-direction, we use Eq. (1.17) with $\omega = 2\pi f = 20\pi$ (rad/s), $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$ (rad/s), $A = 40$ cm, and x assigned a positive sign:

$$y(x,t) = 40\cos\left(20\pi t + \frac{20\pi}{3}x + \phi_0\right) \quad \text{(cm)},$$

with x in meters.
(a) $y(0,0) = 0 = 40\cos\phi_0$. Hence, $\phi_0 = \pm\pi/2$, and

$$\begin{aligned} y(x,t) &= 40\cos\left(20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2}\right) \\ &= \begin{cases} -40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = \pi/2, \\ 40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At $x = 7.5$ cm $= 7.5 \times 10^{-2}$ m, $y = 0 = 40\cos(\pi/2 + \phi_0)$. Hence, $\phi_0 = 0$ or π, and

$$y(x,t) = \begin{cases} 40\cos\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = 0, \\ -40\cos\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm)}, & \text{if } \phi_0 = \pi. \end{cases}$$

Problem 1.12 The voltage of an electromagnetic wave traveling on a transmission line is given by $v(z,t) = 5e^{-\alpha z}\sin(4\pi\times10^9 t - 20\pi z)$ (V), where z is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At $z = 2$ m, the amplitude of the wave was measured to be 1 V. Find α.

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 4\pi\times10^9$ rad/s and $\beta = 20\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 2\times10^9$ Hz $= 2$ GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.1$ m. From Eq. (1.30),

$$u_p = \omega/\beta = 2\times10^8 \text{ m/s}.$$

(b) Using just the amplitude of the wave,

$$1 = 5e^{-\alpha 2}, \qquad \alpha = \frac{-1}{2\text{ m}}\ln\left(\frac{1}{5}\right) = 0.81 \text{ Np/m}.$$

Problem 1.14 Evaluate each of the following complex numbers and express the result in rectangular form:

(a) $z_1 = 4e^{j\pi/3}$,

(b) $z_2 = \sqrt{3}\,e^{j3\pi/4}$,

(c) $z_3 = 6e^{-j\pi/2}$,

(d) $z_4 = j^3$,

(e) $z_5 = j^{-4}$,

(f) $z_6 = (1-j)^3$,

(g) $z_7 = (1-j)^{1/2}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a) $z_1 = 4e^{j\pi/3} = 4(\cos\pi/3 + j\sin\pi/3) = 2.0 + j3.46$.

(b)

$$z_2 = \sqrt{3}e^{j3\pi/4} = \sqrt{3}\left[\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right] = -1.22 + j1.22 = 1.22(-1+j).$$

(c) $z_3 = 6e^{-j\pi/2} = 6[\cos(-\pi/2) - j\sin(-\pi/2)] = -j6$.

(d) $z_4 = j^3 = j\cdot j^2 = -j$, or

$$z_4 = j^3 = (e^{j\pi/2})^3 = e^{j3\pi/2} = \cos(3\pi/2) - j\sin(3\pi/2) = -j.$$

(e) $z_5 = j^{-4} = (e^{j\pi/2})^{-4} = e^{-j2\pi} = 1$.

(f)

$$\begin{aligned} z_6 = (1-j)^3 = (\sqrt{2}e^{-j\pi/4})^3 &= (\sqrt{2})^3 e^{-j3\pi/4} \\ &= (\sqrt{2})^3[\cos(3\pi/4) - j\sin(3\pi/4)] \\ &= -2 - j2 = -2(1+j). \end{aligned}$$

(g)

$$\begin{aligned} z_7 = (1-j)^{1/2} = (\sqrt{2}e^{-j\pi/4})^{1/2} = \pm 2^{1/4}e^{-j\pi/8} &= \pm1.19(0.92 - j0.38) \\ &= \pm(1.10 - j0.45). \end{aligned}$$

Problem 1.15 Complex numbers z_1 and z_2 are given by

$$z_1 = 3 - j2,$$
$$z_2 = \quad 4 \quad j3.$$

(a) Express z_1 and z_2 in polar form.
(b) Find $|z_1|$ by applying Eq. (1.41) and again by applying Eq. (1.43).
(c) Determine the product $z_1 z_2$ in polar form.
(d) Determine the ratio z_1/z_2 in polar form.
(e) Determine z_1^3 in polar form.

Solution:

(a) Using Eq. (1.41),

$$z_1 = 3 \quad j2 = 3.6e^{-j33.7^\circ},$$
$$z_2 = -4 + j3 = 5e^{j143.1^\circ}.$$

(b) By Eq. (1.41) and Eq. (1.43), respectively,

$$|z_1| = |3 \quad j2| = \sqrt{3^2 \quad (\quad 2)^2} = \sqrt{13} = 3.60,$$
$$|z_1| = \sqrt{(3 - j2)(3 + j2)} = \sqrt{13} = 3.60.$$

(c) By applying Eq. (1.47b) to the results of part (a),

$$z_1 z_2 = 3.6e^{-j33.7^\circ} \times 5e^{j143.1^\circ} = 18e^{j109.4^\circ}.$$

(d) By applying Eq. (1.48b) to the results of part (a),

$$\frac{z_1}{z_2} = \frac{3.6e^{-j33.7^\circ}}{5e^{j143.1^\circ}} = 0.72e^{-j176.8}.$$

(e) By applying Eq. (1.49) to the results of part (a),

$$z_1^3 = \left(3.6e^{-j33.7^\circ}\right)^3 = (3.6)^3 e^{-j3\times 33.7^\circ} = 46.66e^{-j101.1^\circ}.$$

Problem 1.21 A voltage source given by $v_s(t) = 25\cos(2\pi \times 10^3 t - 30°)$ (V) is connected to a series RC load as shown in Fig. 1-19. If $R = 1$ MΩ and $C = 200$ pF, obtain an expression for $v_c(t)$, the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

$$\widetilde{V}_c = \widetilde{V}_s \frac{1/j\omega C}{R + 1/j\omega C} = \frac{\widetilde{V}_s}{(1 + j\omega RC)}.$$

Now $\widetilde{V}_s = 25e^{-j30°}$ V with $\omega = 2\pi \times 10^3$ rad/s, so

$$\begin{aligned}\widetilde{V}_c &- \frac{25e^{-j30°}\text{ V}}{1 + j((2\pi \times 10^3 \text{ rad/s}) \times (10^6\ \Omega) \times (200 \times 10^{-12}\text{ F}))} \\ &= \frac{25e^{-j30°}\text{ V}}{1 + j2\pi/5} = 15.57e^{-j81.5°}\text{ V.}\end{aligned}$$

Converting back to an instantaneous value,

$$v_c(t) = \Re e \widetilde{V}_c e^{j\omega t} = \Re e 15.57 e^{j(\omega t - 81.5°)}\text{ V} = 15.57\cos(2\pi \times 10^3 t - 81.5°)\text{ V,}$$

where t is expressed in seconds.

Problem 1.22 Find the phasors of the following time functions:

(a) $v(t) = 3\cos(\omega t \;\;\; \pi/3)$ (V),
(b) $v(t) = 12\sin(\omega t \;\;\; \pi/4)$ (V),
(c) $i(x,t) = 2e^{-3x}\sin(\omega t \;\;\; \pi/6)$ (A),
(d) $i(t) = \;\;\; 2\cos(\omega t \;\;\; 3\pi/4)$ (A),
(e) $i(t) = 4\sin(\omega t \;.\; \pi/3) \;.\; 3\cos(\omega t \;\;\; \pi/6)$ (A).

Solution:

(a) $\widetilde{V} = 3e^{-j\pi/3}$ V.

(b) $v(t) = 12\sin(\omega t \;\;\; \pi/4) = 12\cos(\pi/2 \;\;\; (\omega t \;\;\; \pi/4)) = 12\cos(\omega t \;\;\; \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.

(c)

$$\begin{aligned}i(t) &= 2e^{-3x}\sin(\omega t \;\;\; \pi/6)\text{ A} = 2e^{-3x}\cos(\pi/2 \;\;\; (\omega t \;\;\; \pi/6))\text{ A} \\ &= 2e^{-3x}\cos(\omega t \;\;\; \pi/3)\text{ A,} \\ \widetilde{I} &- 2e^{\;\;3x}e^{\;\;j\pi/3}\text{ A.}\end{aligned}$$

(d)

$$\begin{aligned}i(t) &= \;\;\; 2\cos(\omega t \;\;\; 3\pi/4), \\ \widetilde{I} &= \;\;\; 2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4}\text{ A.}\end{aligned}$$

(e)

$$\begin{aligned}i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\ &= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\ &= 4\cos(\;\;\;\omega t \;\;\; \pi/6) \;\;\; 3\cos(\omega t \;\;\; \pi/6) \\ &= 4\cos(\omega t \;\;\; \pi/6) \;,\; 3\cos(\omega t \;\;\; \pi/6) = 7\cos(\omega t \;\;\; \pi/6), \\ \widetilde{I} &= 7e^{\;\;j\pi/6}\text{ A.}\end{aligned}$$

Problem 1.23 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V),
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V),
(c) $\widetilde{I} = (6 + j8)$ (A),
(d) $\widetilde{I} = -3 + j2$ (A),
(e) $\widetilde{I} = j$ (A),
(f) $\widetilde{I} = 2e^{j\pi/6}$ (A).

Solution:

(a)

$$\widetilde{V} = \quad 5e^{j\pi/3} \text{ V} = 5e^{j(\pi/3-\pi)} \text{ V} = 5e^{-j2\pi/3} \text{ V},$$
$$v(t) = 5\cos(\omega t - 2\pi/3) \text{ V}.$$

(b)

$$\widetilde{V} = j6e^{-j\pi/4} \text{ V} = 6e^{j(-\pi/4+\pi/2)} \text{ V} = 6e^{j\pi/4} \text{ V},$$
$$v(t) = 6\cos(\omega t \;.\; \pi/4) \text{ V}.$$

(c)

$$\widetilde{I} = (6 + j8) \text{ A} = 10e^{j53.1^\circ} \text{ A},$$
$$i(t) = 10\cos(\omega t \quad 53.1^\circ) \text{ A}.$$

(d)

$$\widetilde{I} = \quad 3 \;.\; j2 = 3.61\,e^{j146.31^\circ},$$
$$i(t) = \Re\mathfrak{e}\{3.61\,e^{j146.31^\circ}\,e^{j\omega t}\} = 3.61\cos(\omega t + 146.31 \;) \text{ A}.$$

(e)

$$\widetilde{I} = j = e^{j\pi/2},$$
$$i(t) = \Re\mathfrak{e}\{e^{j\pi/2}e^{j\omega t}\} = \cos(\omega t \quad \pi/2) = \quad \sin\omega t \text{ A}.$$

(f)

$$\widetilde{I} = 2e^{j\pi/6},$$
$$i(t) = \Re\mathfrak{e}\{2e^{j\pi/6}e^{j\omega t}\} = 2\cos(\omega t \;.\; \pi/6) \text{ A}.$$