Homework 12 solutions

Problem 1: 5.30

Problem 5.30 The x–y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2, as shown in Fig. 5-45 (P5.30). If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}}H_{1x} + \hat{\mathbf{y}}H_{1y} + \hat{\mathbf{z}}H_{1z},$$

find:

(a) $\mathbf{H}_2$,

(b) θ_1 and θ_2, and

(c) evaluate $\mathbf{H}_2$, θ_1, and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$.

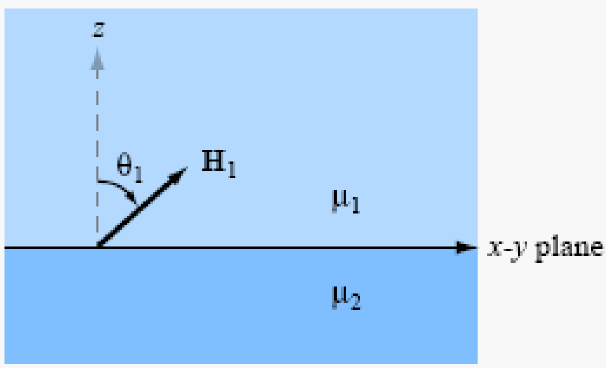

Figure P5.30: Adjacent magnetic media (Problem 5.30).

Solution:

(a) From (5.80),

$$\mu_1 H_{1n} = \mu_2 H_{2n},$$

and in the absence of surface currents at the interface, (5.85) states

$$H_{1t} = H_{2t}.$$

In this case, $H_{1z} = H_{1n}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$
$$H_{1x} = H_{2x},$$
$$H_{1y} = H_{2y},$$

and

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} \frac{\mu_1}{\mu_2} H_{1z}.$$

(b)

$$H_{1t} = \sqrt{H_{1x}^2 \quad H_{1y}^2},$$

$$\tan\theta_1 = \frac{H_{1t}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 \quad H_{1y}^2}}{H_{1z}},$$

$$\tan\theta_2 = \frac{H_{2t}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 \; . \; H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1} \tan\theta_1.$$

(c)

$$\mathbf{H}_2 = \hat{\mathbf{x}}\, 2 \quad \hat{\mathbf{z}} \frac{1}{4} \cdot 4 = \hat{\mathbf{x}}\, 2 \quad \hat{\mathbf{z}} \quad \text{(A/m)},$$

$$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56^\circ,$$

$$\theta_2 = \tan \;\; ^1\left(\frac{2}{1}\right) = 63.44^\circ.$$

Problem 2: 5.32

Problem 5.32 In Fig. 5-46 (P5.32), the plane defined by $x - y = 1$ separates medium 1 of permeability μ_1 from medium 2 of permeability μ_2. If no surface current exists on the boundary and

$$\mathbf{B}_1 = \hat{\mathbf{x}}2 \quad \hat{\mathbf{y}}3 \quad \text{(T)},$$

find $\mathbf{B}_2$ and then evaluate your result for $\mu_1 = 5\mu_2$. Hint: Start out by deriving the equation for the unit vector normal to the given plane.

Solution: We need to find $\hat{\mathbf{n}}_2$. To do so, we start by finding any two vectors in the plane $x - y - 1$, and to do that, we need three non-collinear points in that plane. We choose $(0, -1, 0)$, $(1, 0, 0)$, and $(1, 0, 1)$.

Vector $\mathbf{A}_1$ is from $(0, -1, 0)$ to $(1, 0, 0)$:

$$\mathbf{A}_1 - \hat{\mathbf{x}}\, 1 + \hat{\mathbf{y}}\, 1.$$

Vector $\mathbf{A}_2$ is from $(1, 0, 0)$ to $(1, 0, 1)$:

$$\mathbf{A}_2 = \hat{\mathbf{z}}\, 1.$$

Hence, if we take the cross product $\mathbf{A}_2 \times \mathbf{A}_1$, we end up in a direction normal to the given plane, from medium 2 to medium 1,

$$\hat{\mathbf{n}}_2 = \frac{\mathbf{A}_2 \times \mathbf{A}_1}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{z}}\, 1 \times (\hat{\mathbf{x}}\, 1 \;,\; \hat{\mathbf{y}}\, 1)}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{y}}\, 1 \quad \hat{\mathbf{x}}\, 1}{\sqrt{1+1}} = \frac{\hat{\mathbf{y}}}{\sqrt{2}} \quad \frac{\hat{\mathbf{x}}}{\sqrt{2}}.$$

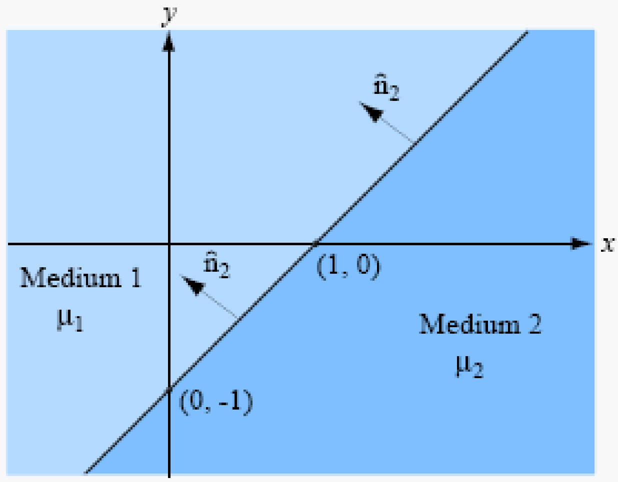

Figure P5.32: Magnetic media separated by the plane $x - y = 1$ (Problem 5.32).

In medium 1, normal component is

$$B_{1n} = \hat{\mathbf{n}}_2 \cdot \mathbf{B}_1 = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\mathbf{B}_{1n} = \hat{\mathbf{n}}_2 B_{1n} = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}.$$

Tangential component is

$$\mathbf{B}_{1t} = \mathbf{B}_1 - \mathbf{B}_{1n} = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) - \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) = \hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5.$$

Boundary conditions:

$$B_{1n} = B_{2n}, \quad \text{or} \quad \mathbf{B}_{2n} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2},$$

$$H_{1t} = H_{2t}, \quad \text{or} \quad \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}.$$

Hence,

$$\mathbf{B}_{2t} = \frac{\mu_2}{\mu_1}\mathbf{B}_{1t} = \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

Finally,

$$\mathbf{B}_2 = \mathbf{B}_{2n} + \mathbf{B}_{2t} = \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) + \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

For $\mu_1 = 5\mu_2$,

$$\mathbf{B}_2 = \hat{\mathbf{y}} \quad \text{(T)}.$$

Problems 3 and 4: see examples 1 and 6 in text book
Problem: 5 6.15

Problem 6.15 A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material with $\varepsilon_r = 9$. The radii of the cylindrical conductors are 0.5 cm and 1 cm. If the voltage applied across the capacitor is

$$V(t) = 50\sin(120\pi t) \quad \text{(V)},$$

what is the displacement current?

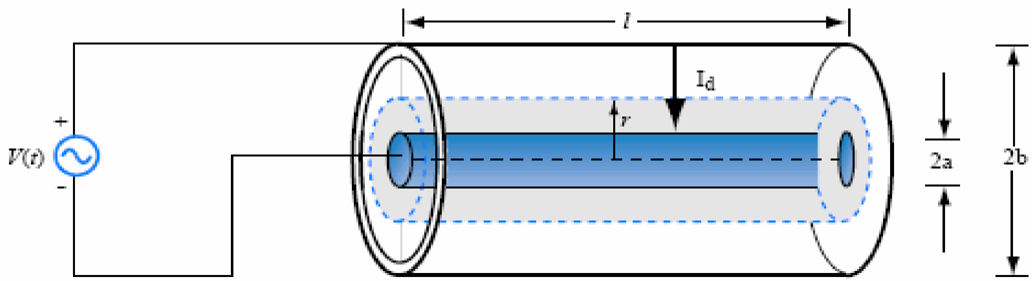

Figure P6.15:

Solution: To find the displacement current, we need to know **E** in the dielectric space between the cylindrical conductors. From Eqs. (4.114) and (4.115),

$$\mathbf{E} = -\hat{\mathbf{r}}\frac{Q}{2\pi\varepsilon r l},$$
$$V = \frac{Q}{2\pi\varepsilon l}\ln\left(\frac{b}{a}\right).$$

Hence,

$$\mathbf{E} = -\hat{\mathbf{r}}\frac{V}{r\ln\left(\frac{b}{a}\right)} = -\hat{\mathbf{r}}\frac{50\sin(120\pi t)}{r\ln 2} = -\hat{\mathbf{r}}\frac{72.1}{r}\sin(120\pi t) \quad \text{(V/m)},$$

$$\begin{aligned}
\mathbf{D} &= \varepsilon\mathbf{E} \\
&= \varepsilon_r\varepsilon_0\mathbf{E} \\
&= -\hat{\mathbf{r}}\,9 \times 8.85 \times 10^{\;12} \times \frac{72.1}{r}\sin(120\pi t) \\
&= \quad \hat{\mathbf{r}}\frac{5.75 \times 10^{-9}}{r}\sin(120\pi t) \quad \text{(C/m}^2\text{)}.
\end{aligned}$$

The displacement current flows between the conductors through an imaginary cylindrical surface of length l and radius r. The current flowing from the outer conductor to the inner conductor along $\quad\hat{\mathbf{r}}$ crosses surface **S** where

$$\mathbf{S} = \quad \hat{\mathbf{r}}\,2\pi r l.$$

Hence,

$$\begin{aligned}
I_d &= \frac{\partial \mathbf{D}}{\partial t}\cdot\mathbf{S} = -\hat{\mathbf{r}}\frac{\partial}{\partial t}\left(\frac{5.75 \times 10^{-9}}{r}\sin(120\pi t)\right)\cdot(-\hat{\mathbf{r}}\,2\pi r l) \\
&= 5.75 \times 10^{-9} \times 120\pi \times 2\pi l\cos(120\pi t) \\
&= 0.82\cos(120\pi t) \quad (\mu\text{A}).
\end{aligned}$$

Alternatively, since the coaxial capacitor is lossless, its displacement current has to be equal to the conduction current flowing through the wires connected to the voltage sources. The capacitance of a coaxial capacitor is given by (4.116) as

$$C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}.$$

The current is

$$I = C\frac{dV}{dt} = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}\left[120\pi \times 50\cos(120\pi t)\right] = 0.82\cos(120\pi t) \quad (\mu\text{A}),$$

which is the same answer we obtained before.