Problem 10.36: A 30 MHz uniform plane wave with

$$H = 10\sin(\omega t + \beta x)\, a_z\, mA/m$$

exists in region $x \geq 0$ having $\sigma = 0, \varepsilon_r = 9, \mu_r = 4$. At x = 0, the wave encounters free space. Determine:

A) The polarization of the wave:

$$a_E \times a_H = a_k \rightarrow a_E \times a_z = -a_x \rightarrow \boldsymbol{a_E = -a_y}$$

B) The phase constant β :

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\pi(30\times10^6)}{3\times10^8}\sqrt{4*9} = \boldsymbol{3.77\, rad/s} = \beta$$

C) The displacement current density in region $x \geq 0$:

$$J_d = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z(x,t) \end{vmatrix} = -\frac{\partial H_z}{\partial x} a_y = -10\beta\cos(\omega t + \beta x)\, a_y$$

$$= \boldsymbol{-37.7\cos(\omega t + \beta x)\, a_y\, mA/m}$$

D) The reflected and transmitted magnetic fields:

$$\eta_2 = \eta_1 = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{2}{3}\eta_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/3}{5/3} = \boldsymbol{1/5} = \Gamma,\ \tau = 1 + \Gamma = \boldsymbol{6/5} = \tau$$

$$E_i = 10\eta_1 \sin(\omega t + \beta x)\, a_E\, mV/m,\ a_E = -a_y$$

Reflected Wave

$$E_r = \Gamma\, 10\eta_1 \sin(\omega t - \beta x)(-a_y) = -\frac{4}{3}\eta_0 \sin(\omega t - \beta x)\, a_y$$

$$a_{E_r} \times a_{H_r} = a_k \rightarrow -a_y \times a_{H_r} = a_x \rightarrow a_{H_r} = -a_z$$

$$H_r = \Gamma\, 10\sin(\omega t - \beta x)(a\, a_z)\, mA/m = \boldsymbol{-2\sin(\omega t - \beta x)\, a_z\, mA/m = H_r}$$

Transmitted Wave

$$E_t = \tau\, 10\eta_1 \sin(\omega t + \beta x)(-a_y)\, mV/m = -8\eta_0 \sin(\omega t + \beta x)\, a_y\, mV/m$$

$$a_{E_t} \times a_{H_t} = a_k \rightarrow -a_y \times a_{H_r} = -a_x \rightarrow a_{H_t} = a_z$$

$$H_t = \tau\, 10\frac{\eta_1}{\eta_2}\sin(\omega t + \beta x)\, a_z\, mA/m = \boldsymbol{8\sin(\omega t + \beta x)\, a_z\, mA/m = H_t}$$

Problem 10.36 Continued

E) The average power density in each region

$$P_{ave1}=\frac{E_{io}^2}{2\eta_1}(-a_x)+\frac{E_{ro}^2}{2\eta_1}(a_x)=-\frac{E_{io}^2}{2\eta_1}\left(1-\Gamma^2\right)a_x=-\frac{\eta_1^2 H_{io}^2}{2\eta_1}a_x$$

$$P_{ave1}=-\frac{\left(\frac{2\eta_0}{3}\right)^2(10^2)}{2\left(\frac{2\eta_0}{3}\right)}(1-\Gamma^2)a_x=\frac{100}{3}\eta_0\left(1-\left(\frac{1}{25}\right)^2\right)\times 10^{-3^2}a_x$$

$$\boldsymbol{P_{ave1}=-12.1\,a_x\,\mu\,W/m}$$

$$P_{ave2}=\frac{E_{ot}^2}{2\eta_2}(-a_x)=\frac{\tau^2\eta_1^2 H_{ot}^2}{2\eta_2}(-a_x)=-\frac{(6/5)^2\left(\frac{2}{3}\eta_0\right)^2(10)^2}{2\eta_0}\times 10^{-3^2}a_x$$

$$\boldsymbol{P_{ave2}=-12.1\,a_x\,\mu\,W/m}$$

Problem 10.37: A uniform plane wave in air is normally incident on an infinite lossless idelectric material having $\varepsilon_r=3\,\mu_r=1$. If the incident wave $E_i=10\cos(\omega t=z)a_y\,V/m$, find:

A) λ and ω of the wave in air and the transmitted wave in the dielectric medium.

$$\beta_1=1,\lambda_1=\frac{2\pi}{\beta_1}=2\pi=\boldsymbol{6.283\,m}=\lambda_{\mathbf{1}}$$

$$\omega_{1,2}=c\beta_1=\boldsymbol{3\times 10^8\,rad/s}=\omega_{\mathbf{1,2}}$$

$$\beta_2=\frac{\omega_2}{c}\sqrt{\mu_r\varepsilon_r}=(1)\sqrt{3}=\boldsymbol{1.73\,rad/m}=\beta_{\mathbf{2}}$$

$$\lambda_2=\frac{2\pi}{\beta_2}=\frac{2\pi}{\sqrt{3}}=\boldsymbol{3.63\,m}=\lambda$$

B) The incident H_i field:

$$H_0=\frac{E_0}{\eta_0}=\frac{10}{120\pi}=0.0265$$

$$a_H=a_k\times a_E\rightarrow a_H=a_z\times a_y\rightarrow a_H=-a_x$$

$$\boldsymbol{H_i=-26.5\cos(\omega t-z)\,a_x\,mA/m}$$

Problem 10.37 Continued

C) Γ and τ :

$$\Gamma=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}=\frac{\frac{1}{\sqrt{3}}-1}{\frac{1}{\sqrt{3}}+1}=-\mathbf{0.268}=\Gamma$$

$$\tau=1+\Gamma=\mathbf{0.732}=\tau$$

D) The total electric field and the time-average power in both regions

$$E_{ot}=\tau E_{io}=7.32, E_{ro}=\Gamma E_{io}=-2.68$$

$$E_1=E_i+E_r=10\cos(\omega t-z)a_y-2.68\cos(\omega t+z)a_y V/m$$

$$E_2=E_t=7.32\cos(\omega t-z)a_y V/m$$

$$P_{ave1}=\frac{(E_{io}^2+E_{ro}^2)}{2\eta_1}a_z=\frac{10^2+(-2.68)^2}{240\pi}a_z=\mathbf{0.1231\,a_z\,W/m^2=P_{ave1}}$$

$$P_{ave2}=\frac{E_{ot}^2}{2\eta_2}a_z=\frac{(\sqrt{3})(7.32)^2}{240\pi}a_z=\mathbf{0.1231\,a_z\,W/m^2=P_{ave2}}$$

Problem 10.45: A plane wave in air with

$$E=(8a_x+6a_y+5a_z)\sin(\omega t+3x-4y)V/m$$

is incident on a copper slab in $y\geq 0$. Find ω and the reflected wave. Asuume copper is a perfect conductor (*Hint:* Write fdown the field components in both media and match the boundary conditions)

$$\beta_1=\sqrt{3^2+4^2}=5=\frac{\omega}{c}\rightarrow\omega=c\beta_1=\mathbf{15\times 10^8\,rad/s}=\omega$$

$$Let\,E_r=(E_0x, E_0y, E_0z)\sin(\omega t+\beta_1 k)a_k V/m$$

We need to find some equations to relate the E_0 coefficients to the coefficients in the hat a_k vector

In a perfect conductor, $\nabla\times E_r=0, thus\, 3E_0x+4E_0y=0$

$$Also, at\, y=0, E_1\tan=E_2\tan=0$$

$$E_1\tan=0\rightarrow 8a_x+6a_y+5a_z+E_0x\,a_x+E_0z\,a_z=0$$

$$By\,separating\,variables, we\,arrive\,at\,E_0x=-8\text{ and }E_0z=-5$$

$$Using\,the\,first\,equation\,above\,that\,relates\,E_0x\text{ and }E_0y,$$

$$we\,have\,E_0y=-3/4\,E_0x=-0.75*(-8)=6$$

$$Hence, \mathbf{E_r=(-8a_x+6a_y-5a_z)\sin(\omega t+3x+4y)\,V/m}$$

Problem 10.46: A polarized wave is incident from air to polystyrene with $\mu_{r2}=1, \varepsilon_{r2}=2.6$ at Brewster angle. Determine the transmission angle.

We know to use the parallel equations because the “polarized wave” is “incident,” thus the wave is polarized to the angle of incidence.

$$\tan\theta_{B\|}=\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}=\sqrt{2.6}=1.61\rightarrow\theta_{B\|}=\tan^{-1}(1.61)=\mathbf{58.11^o}=\theta_{\boldsymbol{B}\|}$$

$$\cos\theta_t=\frac{\eta_1}{\eta_2}\cos\theta_{B\|}=\frac{\eta_0}{\eta_0/\sqrt{2.6}}\cos(58.11^o)=0.8499$$

$$\theta_t=\cos^{-1}(0.8499)=\mathbf{31.89^o}=\theta_{\boldsymbol{t}}$$

Problem 1: For the plane wave given by:

$$E_1(z)=4\cos(\omega t-\beta z)\mathbf{a}_x+2\cos(\omega t+90^o-\beta z)\mathbf{a}_y$$

Determine the following

1) Direction of propagation

This plane wave is propagating in the $+\mathbf{a}_z$ direction

2) The polarization

This can be rewritten as the following (to more easily follow the solution to Problem 3 in HW#1)

$$E=E_1+E_2=4e^{-j\beta z}\mathbf{a}_x+2e^{-j\beta z}e^{j\frac{\pi}{2}}\mathbf{a}_y$$

Notice this has the phase shift $(\pi/2)$ present in the elliptical polarization solution. Thus we can arrive at the following two relations for $E_1 \text{ and } E_2$.

$$\frac{E_x^2}{4^2}=\cos^2(\omega t-\beta z)\mathbf{a}_x$$

$$\frac{E_y^2}{2^2}=(-\sin(\omega t-\beta z))^2\mathbf{a}_y=\sin^2(\omega t-\beta z)\mathbf{a}_y$$

Thus we have an elliptical polarization defined by the following equation.

$$\frac{\boldsymbol{E}_x^2}{\mathbf{16}}+\frac{\boldsymbol{E}_y^2}{\mathbf{4}}=\mathbf{1}$$