Problem 10.26: A uniform plane wave in a lossy nonmagnetic media has

$$E_s=(5\hat{a}_x+12\hat{a}_y)e^{-\gamma z}, \text{ where } \gamma=0.2+j3.4$$

A) Compute the magnitde of the wave at z = 4m.

$$E_S=\Re\left[E_s e^{j\omega t}\right]=(5\hat{a}_x+12\hat{a}_y)e^{-\gamma z}, \gamma=\alpha+j\beta=0.2+j3.4$$

$$note: \left|5\hat{a}_x+12\hat{a}_y\right|=\sqrt{5^2+12^2}=13$$

$$|E|=13e^{-(0.2)*4}\left|\cos(\omega t-3.4z)\right|=13e^{-0.8}(1)=\mathbf{5.84}=|\boldsymbol{E}|$$

B) Find the loss in dB suffered by the wave in the interval 0 < z < 3m

$$Loss=\alpha(\Delta z)=(0.2)(3m)=0.6\,Np, 1\,Np=8.686\,dB,$$

$$Loss=(0.6)(8.686)=\mathbf{5.212\,dB}=\boldsymbol{Loss}$$

C) Calculate the Poynting vector at z = 4,t = T/8. Take $\omega=10^8$ rad/s

$$Let\ x=\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}\ then\ \frac{\alpha}{\beta}=\sqrt{\frac{x-1}{x+1}}=\frac{0.2}{3.4} \quad now\ solve\ for\ x$$

$$x-1=\frac{1}{17^2}(x+1) \quad\rightarrow\quad x=\frac{1+1/17^2}{1-1/17^2}=1.00694=x$$

$$\alpha=\omega\sqrt{\frac{\varepsilon\mu}{2}(x-1)}=\frac{\omega}{c}\sqrt{\frac{\varepsilon_r\mu_r}{2}(x-1)} \quad \text{Use a little algebra to solve for } \varepsilon_r$$

$$\varepsilon_r=\left(\frac{\alpha c}{\omega}\right)^2*\frac{2}{x-1}=\left(\frac{(0.2)(3\times10^8)}{10^8}\right)^2*\left(\frac{2}{1.00694-1}\right)=103.7$$

$$|\eta|=\frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}}{\sqrt{x}}=\frac{120\pi}{\sqrt{103.7*1.00694}}=36.88$$

$$\tan 2\theta_\eta=\frac{\sigma}{\mu\varepsilon}=\sqrt{x^2-1}=0.118\rightarrow\theta_\eta=\tan^{-1}0.118=3.365^o$$

$$H_s=\hat{a}_k\times\frac{E_s}{\eta}=\frac{\hat{a}_z}{\eta}\times(5\hat{a}_x+12\hat{a}_y)e^{-\gamma z}=\frac{(5\hat{a}_x+12\hat{a}_y)}{|\eta|}e^{-j3.365^o}e^{-\gamma z}$$

$$H_s=(-369.2\hat{a}_x+153.8\hat{a}_y)\cos(\omega t-2.4z-3.365^o)\,mA/m$$

$$P=E\times H=\begin{bmatrix}5 & 12 & 0\\ -0.369 & 0.153 & 0\end{bmatrix}\times e^{-0.4z}\cos(\omega t-3.4z)\cos(\omega t-3.4z-3.365^o)\hat{a}_z$$

$$P=5.3e^{-1.6}\cos(\pi/4-13.6)\cos(\pi/4-13.6-0.0587)\hat{a}_z=\mathbf{0.4613\,\hat{a}_z\,W/m^2}=\boldsymbol{P}$$

Problem 10.29: In a transmission line filled with a lossless dielectric

$(\varepsilon_r = 4.5, \mu_r = 1)$

$$E = \frac{40}{\rho}\sin(\omega t - 2z)\hat{a}_\rho V/m$$

A) Find ω and H

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} \rightarrow \omega = \frac{c\beta}{\sqrt{\mu_r\varepsilon_r}} = 2\frac{(3\times10^8)}{\sqrt{4.5}} = \mathbf{282.8\times10^6}\, \boldsymbol{rad/s} = \omega$$

$$H = \frac{E}{\eta} = \frac{\hat{a}_z}{\eta}\times\frac{40}{\rho}\sin(\omega t - 2z)\hat{a}_\phi = \frac{\mathbf{0.225}}{\rho}\mathbf{sin}(\omega \boldsymbol{t} - \mathbf{2}\boldsymbol{z})\hat{\boldsymbol{a}}_\phi \boldsymbol{A/m} = \boldsymbol{H}$$

B) The Poynting vector

$$P = E\times H = \frac{\mathbf{9}}{\rho^{\mathbf{2}}}\mathbf{sin^2}(\omega \boldsymbol{t} - \mathbf{2}\boldsymbol{z})\hat{\boldsymbol{a}}_z \boldsymbol{W/m^2} = \boldsymbol{P}$$

C) The time-average power crossing the surface of

$$(x = 1, 2\,mm < \rho < 3\,mm, 0 < \phi < 2\pi)$$

$$P_{ave} = \frac{4.5}{\rho^2}\hat{a}_z, dS = \rho\, d\,\phi\, d\,\rho\,\hat{a} - z$$

$$P_{ave-time} = \oint P_{ave}\, dS = 4.5\int_{2mm}^{3mm}\frac{d\rho}{\rho}\int_0^{2\pi} d\,\phi\,\hat{a}_z = (4.5)(\ln\frac{3}{2})(2\pi)$$

$$\boldsymbol{P_{ave-time}} = \mathbf{11.45}\,\boldsymbol{W}$$

Problem 10.31: The plane wave $E=30\cos(\omega t-z)\hat{z}_x$ V/m in air normally hits a lossless medium $(\mu_r=1,\varepsilon_r=4)$ at z = 0.

$$note: \eta_1=\eta_0=120\pi\,,\ \eta_2=0.5\,\eta_0=60\pi$$

A) Find Γ, τ, and s

$$\Gamma=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}=\frac{60\pi-120\pi}{60\pi+120\pi}=\mathbf{-1/3}=\Gamma$$

$$\tau=1+\Gamma=\mathbf{2/3}=\tau$$

$$s=\frac{1+|\Gamma|}{1-|\Gamma|}=\frac{1+1/3}{1-1/3}=\mathbf{2}=\boldsymbol{s}$$

B) Calculate the reflected electric and magnetic fields.

$$E_{ro}=E_{io}\Gamma=-1/3*30=-10$$

$$E_r=-10\cos(\omega t-z)\hat{a}_x\, v/m$$

$$H_r=H_{ro}\cos(\omega t+z)\hat{a}_H$$

$$\hat{a}_E\times\hat{a}_H=\hat{a}_k\rightarrow\hat{a}_k\times\hat{a}_H=-\hat{a}_z\rightarrow\hat{a}_H=\hat{a}_y$$

$$H_r=\frac{E_{ro}}{|\eta_0|}\cos(\omega t+z)\hat{a}_y=\frac{-10}{120}\pi\cos(\omega t+z)\hat{a}_y$$

$$\boldsymbol{H_R=26.53\cos(\omega t+z)\hat{a}_y\, mA/m}$$

Problem 10.32: A uniform plane wave in air with $H = 4\sin(\omega t - 5x)\hat{a}_y$ A/m is normally incident on a plastic region with parameters $\mu_r = 1, \varepsilon = 4 \text{ and } \sigma = 0$.

$$note: \eta_1 = \eta_0 = 120\pi\,,\ \eta_2 = 0.5\,\eta_0 = 60\pi$$

A) Obtain the total electric field in air

$$E_i = E_{oi}\sin(\omega t - 5x)\hat{a}_E$$
$$E_{io} H_{io}\eta_0 = 4 * 120\pi = 480\pi$$
$$\hat{a}_E \times \hat{a}_H = \hat{a}_k \rightarrow \hat{a}_E \times \hat{a}_y = \hat{a}_x \rightarrow a_E = -\hat{a}_z$$
$$E_i = -480\pi\sin(\omega t - 5x)\hat{a}_z$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \mathbf{-1/3} = \Gamma$$
$$\tau = 1 + \Gamma = \mathbf{2/3} = \tau$$

$$E_{ro} = \Gamma\, E_{io}(-1/3)(480\pi) = 160\pi$$
$$E_r = 160\pi\sin(\omega t + 5x)\hat{a}_z$$

$$E = E_i + E_r = -480\pi\sin(\omega t - 5x)\hat{a}_z + 160\pi\sin(\omega t + 5x)\hat{a}_z$$
$$\boldsymbol{E = -1508\sin(\omega t - 5x)\hat{a}_z + 503\sin(\omega t + 5x)\hat{a}_z\, V/m}$$

B) Calculate the time-average power density in the lastic region

$$E_{ot} = \tau\, E_{io} = (2/3)(480\pi) = 320\pi$$
$$P_{ave} = \frac{E_{ot}^2}{2\eta_2}\hat{a}_x = \frac{(320\pi)^2}{120\pi}\hat{a}_x = 853\pi = \mathbf{2.68\, kW/m^2} = \boldsymbol{P_{ave}}$$

C) Find the standing wave ratio

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3} = \mathbf{2} = \boldsymbol{s}$$

Problem 1: A uniform plane wave emitted by a communication repeater is propagating in a lossless dielectric medium along the positive z direction. The time-domain form of the electric field (in V/m) is given by:

$$E(z,t)=\hat{a}_x 377\cos\left(\omega t-\frac{4\pi}{3}+\frac{\pi}{6}\right)V/m$$

The time-average power density of the emitted wave is $377\,W/m^2$. Determine the following:

A) Properties of the popagation medium assuming $\mu=\mu_0 (i.e.\,\eta,\varepsilon)$

$$We\,are\in a\,lossless\,medium\,so \quad \sigma=0$$

$$\eta=\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}$$

$$P_{ave}=\frac{1}{2}\frac{E_m^2}{\eta}=377\,W/m^2\rightarrow\eta=\frac{377^2}{2*377}=\frac{377}{2}=\mathbf{188.5}=\eta$$

$$\eta=\frac{377}{2}=\frac{377}{\sqrt{\varepsilon_r}}\rightarrow\varepsilon_r=\mathbf{4}$$

B) Frequency of the wave

$$\beta=\frac{4\pi}{3}=\omega\sqrt{\varepsilon_0\mu_0\varepsilon_r}=\frac{\omega}{c}\sqrt{\varepsilon_r}\rightarrow\omega=c\frac{\beta}{\sqrt{\varepsilon_r}}=\frac{4\pi}{3}\frac{3\times10^8}{2}=2\pi\times10^8\,rad/s$$

$$f=\frac{\omega}{2\pi}=\frac{2\pi\times10^8}{2\pi}=10^8\,Hz=\mathbf{100\,MHz}=\boldsymbol{f}$$

C) Expression for the time-domain form of the magnetic field

$$H(z,t)=\frac{377}{\eta}\cos\left(2\pi\times10^8 t-\frac{4\pi}{3}z+\frac{\pi}{6}\right)\hat{a}_y$$

$$¿\,\mathbf{2\cos\left(2\pi\times10^8 t-\frac{4\pi}{3}z+\frac{\pi}{6}\right)\hat{a}_y\,A/m}$$

Problem 2: A uniform plane wave is propagating in a lossless dielectric medium along the positive z direction. The phasor form of the electric field (in V/m) is given by:

$$E_s = \hat{a}_x \left(40\pi e^{j\frac{4\pi}{6}}\right) e^{-j\frac{4\pi}{3}z} \; V/m$$

The time-average power density for this wave is $377\,W/m^2$

D) If $\mu = \mu_0$ determine the relative dielectric constant of the medium ε_r

$We\,are \in a\,lossless\,medium\,so \quad \sigma = 0$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}$$

$$P_{ave} = \frac{1}{2}\frac{E_m^2}{\eta} = 377\,W/m^2 \rightarrow \eta = \frac{(40\pi)^2}{2*377} = 20.9 = \eta$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \rightarrow \varepsilon_r = \frac{(120\pi)^2}{20.9^2} = \mathbf{325}$$

E) The frequency of the wave

$$\beta = \frac{4\pi}{3} = \omega\sqrt{\varepsilon_0 \mu_0 \varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r} \rightarrow \omega = c\frac{\beta}{\sqrt{\varepsilon_r}} = \frac{4\pi}{3}\frac{3\times 10^8}{\sqrt{325}} = 69.7\,Mrad/s$$

$$f = \frac{\omega}{2\pi} = \frac{69.7\times 10^6}{2\pi} = \mathbf{11.1\,MHz = f}$$

F) The expression for the phasor form of the magnetic field.

$$H = \frac{40\pi}{\eta}\cos(\omega t - \frac{4\pi}{3}z + \frac{4\pi}{6})\hat{a}_y = \mathbf{6\cos(\omega t - \frac{4\pi}{3}z + \frac{4\pi}{6})\hat{a}_y = 6e^{\frac{-4\pi}{3}z} e^{\frac{4\pi}{6}}\hat{a}_y\,A/m}$$

Problem 3: A plane wave is incident normal to the surface of sea water $(\mu_r = 1, \varepsilon_r = 79, \sigma = 3\,S/m)$. The electric field is parallel to the surface and its magnitude is 10 V/mjust inside the surface of the water (at air-water interface). At the following frequencies: (i) 20kHz (ii) 20GHz, calculate the depth at which a submarine will be able to receive a signal if the receivers on board the submarine require a minimum field intensity of $10\,\mu V/m$.

i) 20kHz

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_r}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}$$

$$\alpha = \frac{2\pi\,20\times10^3}{3\times10^8}\sqrt{\frac{79}{2}\left(\sqrt{1+\left(\frac{3}{(2\pi*20\times10^3)(79)(8.854\times10^{-12})}\right)^2}-1\right)} = 0.486$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_r}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

$$\beta = \frac{2\pi\,20\times10^3}{3\times10^8}\sqrt{\frac{79}{2}\left(\sqrt{1+\left(\frac{3}{(2\pi*20\times10^3)(79)(8.854\times10^{-12})}\right)^2}+1\right)} = 0.487$$

$$E = 10\,e^{-\gamma z} = 10\,e^{-\alpha z}\,e^{-j\beta z} \quad \text{solve for z when } E = 10\,\mu V$$

$$10\times10^{-6} = 10\,e^{-\gamma z} \rightarrow z = \frac{\ln(10^{-6})}{|\gamma|} = \frac{\ln(10^{-6})}{\sqrt{\alpha^2+\beta^2}} = \ln\frac{(10^{-6})}{\sqrt{0.486^2+0.487^2}} = -20.1\,m$$

The submarine can be up to 20.1 meters below the surface when communicating at 20kHz

Problem 3 (continued)

ii) 20GHz

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}=\frac{\omega}{c}\sqrt{\frac{\varepsilon_r}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}$$

$$\alpha=\frac{2\pi\, 20\times10^9}{3\times10^8}\sqrt{\frac{79}{2}\left(\sqrt{1+\left(\frac{3}{(2\pi*20\times10^9)(79)(8.854\times10^{-12})}\right)^2}-1\right)}=2215$$

$$\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}=\frac{\omega}{c}\sqrt{\frac{\varepsilon_r}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

$$\beta=\frac{2\pi\, 20\times10^9}{3\times10^8}\sqrt{\frac{79}{2}\left(\sqrt{1+\left(\frac{3}{(2\pi*20\times10^9)(79)(8.854\times10^{-12})}\right)^2}+1\right)}=3052$$

$$E=10\,e^{-\gamma z}=10\,e^{-\alpha z}\,e^{-j\beta z}\quad \text{solve for z when } E=10\,\mu V$$

$$10\times10^{-6}=10\,e^{-\gamma z}\rightarrow z=\frac{\ln(10^{-6})}{|\gamma|}=\frac{\ln(10^{-6})}{\sqrt{\alpha^2+\beta^2}}=\ln\frac{(10^{-6})}{\sqrt{2215^2+3052^2}}=-3.66\,mm$$

The submarine can be up to 3.66 millimeters below the surface when communicating at 20GHz