Problem 10.43b: A parallel-polarized wave in air with
\[ E = (8a_y - 6a_z) \sin(\omega t - 4y - 3z) \text{V/m} \]
impinges a dielectric half-space as shown in Figure 10.21. Find:

b) The time-average power in air \((\mu = \mu_0 \text{ and } \varepsilon = \varepsilon_0)\).
\[
P_{\text{ave}} = \frac{E_0^2}{2\eta} a_k = \frac{\sqrt{8^2 + 6^2}}{2*120\pi} \frac{4a_y + 3a_z}{5}
\]
\[
P_{\text{ave}} = 106.1a_y + 79.58a_z \text{mW/m}^2
\]

Problem 12.1: A parallel-polarized wave in air with

A) Show that a rectangular waveguide does not support \(\text{TM}_{10}\) and \(\text{TM}_{01}\)

For \(\text{TM}_{mn}\) modes, \(H_z = 0\) thus \(E_z = E_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi x}{b}\right) e^{-yz}\)

For \(\text{TM}_{01}\), \(a = 0\) thus the \(\sin\left(\frac{\pi x}{a}\right)\) term is undefined.

For \(\text{TM}_{10}\), \(b = 0\) thus the \(\sin\left(\frac{\pi x}{b}\right)\) term is undefined.

Because of these undefined terms, \(\text{TM}_{01}\) and \(\text{TM}_{10}\) are not supported by a rectangular waveguide

B) Explain the difference between \(\text{TE}_{mn}\) and \(\text{TM}_{mn}\) modes

In TM modes, the magnetic field has its components transverse (or normal) to the direction of wave propagation. This implies that we set \(H_z = 0\).

In TE modes, the electric field is transverse (or normal) to the direction of wave propagation. We set \(E_z = 0\).
Problem 12.2: A 2-cm by 3cm waveguide is filled with a dielectric material with \( \varepsilon_r=4 \). If the guide operates at 20GHz with \( TM_{11} \) mode, find:

A) cutoff frequency

\[
\begin{align*}
\nu' &= \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8 \\
\nu &= \frac{\nu'}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{1.5 \times 10^8}{2 \times 10^{-2}} \sqrt{\frac{1}{2^2} + \frac{1}{3^2}} = 4.507 \text{GHz} = f_c
\end{align*}
\]

B) the phase constant

\[
\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{\omega}{\nu} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{2 \pi 20 \times 10^9}{3 \times 10^8} \sqrt{1 - \left(\frac{4.507}{20}\right)^2} \\
\beta = 816.2 \text{rad/m}
\]

C) the phase velocity

\[
\nu = \frac{\omega}{\beta} = \frac{2 \pi 20 \times 10^9}{816.2} = 1.54 \times 10^8 \text{m/s} = u
\]

Problem 12.6: In an air-filled rectangular waveguide, the cutoff frequency of a \( TE_{10} \) mode is 5GHz, whereas that of \( TE_{01} \) is 12GHz. Calculate:

Recall \( f_c = \frac{u}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} \)

A) The dimensions of the guide

\[
\begin{align*}
\nu &= \frac{u'}{2} \sqrt{\frac{m^2}{a^2}} = 5 \times 10^9 \Rightarrow a = \frac{u'}{2 f_{c_{10}}} = \frac{3 \times 10^8}{2 \times 5 \times 10^9} = 3.00 \text{cm} = a \\
\nu &= \frac{u'}{2} \sqrt{\frac{n^2}{b^2}} = 12 \times 10^9 \Rightarrow b = \frac{u'}{2 f_{c_{01}}} = \frac{3 \times 10^8}{2 \times 12 \times 10^9} = 1.25 \text{cm} = b
\end{align*}
\]
Problem 12.6 Continued:

B) The cutoff frequencies of the next three higher TE modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cutoff Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{10}$</td>
<td>5 GHz</td>
</tr>
<tr>
<td>* $TE_{20}$</td>
<td>10 GHz</td>
</tr>
<tr>
<td>$TE_{30}$</td>
<td>15 GHz</td>
</tr>
<tr>
<td>$TE_{40}$</td>
<td>20 GHz</td>
</tr>
<tr>
<td>* $TE_{01}$</td>
<td>12 GHz</td>
</tr>
<tr>
<td>$TE_{02}$</td>
<td>24 GHz</td>
</tr>
<tr>
<td>* $TE_{11}$</td>
<td>13 GHz</td>
</tr>
<tr>
<td>$TE_{21}$</td>
<td>15.6 GHz</td>
</tr>
</tbody>
</table>

The three next higher modes are: $TE_{20}, TE_{01}$ and $TE_{11}$.

C) The cutoff frequency for $TE_{11}$ mode if the guide is filled with a lossless material having $\varepsilon_r = 2.25$ and $\mu_r = 1$.

\[
 f_{c_{11}} = \frac{u}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} = \frac{3 \times 10^8}{2 \times 10^{-2} \sqrt{2.25}} \sqrt{\frac{1^2}{3^2} + \frac{1^2}{1.25^2}} = 8.67 \text{ GHz} = f_{c_{11}}
\]
Problem 1: A plane wave is normally incident on a planar interface between air and muscle. The conductivity and dielectric constants of muscle are \( \sigma_m = 0.889 \text{ S/m} \) and \( \epsilon_r = 71.7 \). The intrinsic impedance of a conducting (or lossy) medium such as muscle or fat is given by:

\[
\eta_c = \sqrt{\frac{\mu}{\epsilon}} e^{j\frac{\pi}{4} \tan^{-1} \left( \frac{\sigma}{\omega \epsilon} \right)}
\]

Determine the percentage of incident power absorbed by the muscle tissue at:

a) 100 Mhz
\[
\eta_c = 28.5 e^{j33.01} = 23.9 + j 15.5
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{23.9 + j 15.5 - 377}{23.9 + j 15.5 + 377} = -353 e^{j177} \approx 0.878 e^{j175}
\]
\[
\left| \frac{(P_{av})_i}{(P_{av})_t} \right| = (1 - \rho^2) = (1 - 0.878^2) = 0.229 = 22.9\%
\]

b) 300 Mhz
\[
\eta_c = 40.0 e^{j18.4} = 37.9 + j 12.6
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{37.9 + j 12.6 - 377}{37.9 + j 12.6 + 377} = -339 e^{j176} \approx 0.817 e^{j176}
\]
\[
\left| \frac{(P_{av})_i}{(P_{av})_t} \right| = (1 - \rho^2) = (1 - 0.817^2) = 0.332 = 33.2\%
\]

c) 915 Mhz
\[
\eta_c = 44.1 e^{j6.9^\circ} = 43.7 + j 5.29
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{43.7 + j 5.29 - 377}{43.7 + j 5.29 + 377} = -333 e^{j179^\circ} \approx 0.792 e^{j178^\circ}
\]
\[
\left| \frac{(P_{av})_i}{(P_{av})_t} \right| = (1 - \rho^2) = (1 - 0.792^2) = 0.373 = 37.3\%
\]

d) 2.45 Ghz.
\[
\eta_c = 44.6 e^{j2.62^\circ} = 44.6 + j 2.04
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.6 + j 2.04 - 377}{44.6 + j 2.04 + 377} = -332 e^{j180^\circ} \approx 0.789 e^{j180^\circ}
\]
\[
\left| \frac{(P_{av})_i}{(P_{av})_t} \right| = (1 - \rho^2) = (1 - 0.789^2) = 0.378 = 37.8\%
\]
Problem 1 Continued

Repeat (a) - (d) if muscle is replaced by Fat \((\sigma_f = 0.155 \, S/m \text{ and } \varepsilon_r = 71.7)\)

a) 100 Mhz
\[
\eta_f = 43.1 e^{j10.6^\circ} = 42.4 + j 8.01
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{42.4 + j 8.01 - 377}{42.4 + j 8.01 + 377} = \frac{335 e^{j179^\circ}}{419 e^{j1.09^\circ}} = 0.798 e^{j178^\circ}
\]
\[
\left|\left(\frac{P_{av}}{P_{av}}\right)\right| = (1 - \rho^2) = (1 - 0.798^2) = 0.363 = 36.6\%
\]

b) 300 Mhz
\[
\eta_f = 44.5 e^{j3.72^\circ} = 44.4 + j 2.89
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.4 + j 2.89 - 377}{44.4 + j 2.89 + 377} = \frac{333 e^{j179.5^\circ}}{421 e^{j0.39^\circ}} = 0.789 e^{j178^\circ}
\]
\[
\left|\left(\frac{P_{av}}{P_{av}}\right)\right| = (1 - \rho^2) = (1 - 0.789^2) = 0.377 = 37.7\%
\]

c) 915 Mhz
\[
\eta_f = 44.7 e^{j1.23^\circ} = 44.7 + j 0.96
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.7 + j 0.96 - 377}{44.7 + j 0.96 + 377} = \frac{332 e^{j180^\circ}}{422 e^{j0.13^\circ}} = 0.788 e^{j180^\circ}
\]
\[
\left|\left(\frac{P_{av}}{P_{av}}\right)\right| = (1 - \rho^2) = (1 - 0.788^2) = 0.379 = 37.9\%
\]

d) 2.45 Ghz.
\[
\eta_f = 44.7 e^{j0.46^\circ} = 44.7 + j 0.36
\]
\[
\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.7 + j 0.36 - 377}{44.7 + j 0.26 + 377} = \frac{332 e^{j180^\circ}}{422 e^{j0.05^\circ}} = 0.788 e^{j180^\circ}
\]
\[
\left|\left(\frac{P_{av}}{P_{av}}\right)\right| = (1 - \rho^2) = (1 - 0.788^2) = 0.379 = 37.9\%
Problem 2: The electric field associated with a uniform plane wave propagating in air is given by: \( E_i = 1000 \cos \left( \frac{10^8 \pi t - \beta_0 z}{c} \right) \alpha \) V/m. The wave is normally incident on a dielectric medium \((\sigma_d = 0, \varepsilon_d = 5 \varepsilon_0, \mu_d = \mu_0)\)

Determine the following:

(a) \( \beta_0 \) in air and \( \beta_d \) in dielectric material.

\[
\beta_0 = \frac{\omega}{c} = \frac{10^8 \pi}{3 \times 10^8} = \frac{\pi}{3} = 1.047 \text{ rad/m} = \beta_0
\]

\[
\beta_d = \frac{\omega}{c} \sqrt{\mu \varepsilon_r} = \frac{10^8 \pi}{3 \times 10^8} \sqrt{\frac{1}{5}} = 2.342 \text{ rad/m} = \beta_d
\]

(b) Reflection and transmission coefficients

\[
\eta_2 = \eta_0 \sqrt{\frac{\mu_r \varepsilon_r}{\varepsilon_r}} = \eta_0 \sqrt{\frac{1}{5}}, \quad \eta_1 = \eta_0
\]

\[
\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \sqrt{\frac{1}{5}} = -0.382 = \Gamma
\]

\[
\tau = 1 + \Gamma = 1 - 0.382 = 0.618 = \tau
\]

(c) Amplitudes of the reflected and transmitted E and H fields.

\[
|E_{ot}| = |\Gamma| |E_0| = 0.382 \times 1000 = 382 = |E_{ot}|, \quad |H_{ot}| = \frac{E_{ot}}{\eta_2} = \frac{618}{120 \pi \sqrt{\frac{1}{5}}} = 3.67 = |H_{ot}|
\]

\[
|E_{or}| = |\eta_2| |E_0| = 0.618 \times 1000 = 618 = |E_{or}|, \quad |H_{or}| = \frac{E_{or}}{\eta_1} = \frac{318}{120 \pi \sqrt{\frac{1}{5}}} = 1.01 = |H_{or}|
\]

(d) Reflected and transmitted power

\[
P_{ave} = \frac{E_{ot}^2}{2 \eta_2} = \frac{618^2}{240 \pi \sqrt{\frac{1}{5}}} = 1.13 kW/m^2 = P_{ave}
\]

\[
P_{ave} = \frac{E_{or}^2}{2 \eta_1} = \frac{318^2}{240 \pi \sqrt{\frac{1}{5}}} = 194 kW/m^2 = P_{ave}
\]