Problem 10.43b: A parallel-polarized wave in air with

$E=(8a_y-6a_z)\sin(\omega t-4y-3z)\,V/m$ impinges a dielectric half-space as shown in Figure 10.21. Find:

b) The time-average power in air $(\mu=\mu_0 \text{ and } \varepsilon=\varepsilon_0)$.

$$P_{ave}=\frac{E_0^2}{2\eta}a_k=\frac{\sqrt{8^2+6^2}}{2*120\pi}*\frac{4a_y+3a_z}{5}$$

$$\boldsymbol{P_{ave}=106.1\,a_y+79.58\,a_z\,mW/m^2}$$

Problem 12.1: A A parallel=polarized wave in air with

A) Show that a rectangular waveguide does not support $TM_{10}\,\text{and}\,TM_{01}$

For TM_{mn} modes, $H_z=0$ thus $E_{sz}=E_0\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi x}{b}\right)e^{-\gamma z}$

For TM_{01}, $a=0$ thus the $\sin\left(\frac{\pi x}{a}\right)$ term is undefined.

For TM_{10}, $b=0$ thus the $\sin\left(\frac{\pi x}{b}\right)$ term is undefined.

Because of these undefined terms, $TM_{01}\,\text{and}\,TM_{10}$ are not supported by a rectangular waveguide

B) Explain the difference between $TE_{mn}\,\text{and}\,TM_{mn}$ modes

In TM modes, the magnetic field has its components transverse (or normal) to the direction of wave propagation. This implies that we set $H_z=0$.

In TE modes, the electric field is transverse (or normal) to the diection of wave propagation. We set $E_z=0$.

Problem 12.2: A 2-cm by 3cm waveguide is filled with a dielectric material with $\varepsilon_r = 4$. If the guide operates at 20GHz with TM_{11} mode, find:

A) cutoff frequency

$$u' = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{3\times10^8}{2} = 1.5\times10^8$$

$$f_c = \frac{u'}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{1.5\times10^8}{2\times10^{-2}}\sqrt{\frac{1}{2^2}+\frac{1}{3^2}} = \mathbf{4.507\,GHz = f_c}$$

B) the phase constant

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{\omega}{u'}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi\, 20\times10^9}{3\times10^8}\sqrt{1-\left(\frac{4.507}{20}\right)^2}$$

$$\beta = \mathbf{816.2\, rad/m}$$

C) the phase velocity

$$u = \frac{\omega}{\beta} = \frac{2\pi * 20\times10^9}{816.2} = \mathbf{1.54\times10^8\, m/s = u}$$

Problem 12.6: In an air-filled rectangular waveguide, the cutoff frequency of a TE_{10} mode is 5GHz, whereas that of TE_{01} is 12GHz. Calculate:

Recall $f_c = \frac{u}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$

A) The dimensions of the guide

$$f_{c_{10}} = \frac{u'}{2}\sqrt{\frac{m^2}{a^2}} = 5\times10^9 \rightarrow a = \frac{u'}{2f_{c_{10}}} = \frac{3\times10^8}{2*5\times10^9} = \mathbf{3.00\, cm = a}$$

$$f_{c_{10}} = \frac{u'}{2}\sqrt{\frac{n^2}{b^2}} = 12\times10^9 \rightarrow b = \frac{u'}{2f_{c_{10}}} = \frac{3\times10^8}{2*12\times10^9} = \mathbf{1.25\, cm = b}$$

Problem 12.6 Continued:

B) The cutoff frequencies of the next three higher TE modes

Mode	***Cutoff Frequency***
TE_{10}	5 GHz
* TE_{20}	10 Ghz
TE_{30}	15 GHz
TE_{40}	20 GHz
* TE_{01}	12 GHz
TE_{02}	24 GHz
* TE_{11}	13 GHz
TE_{21}	15.6 GHz

The three next higher modes are: $\boldsymbol{TE_{20}, TE_{01}}$ **and** $\boldsymbol{TE_{11}}$**.**

C) The cutoff frequency for TE_{11} mode if the guide is filled with a lossless material having $\varepsilon_r = 2.25$ and $\mu_r = 1$.

$$f_{c_{11}} = \frac{u}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}} = \frac{3\times10^8}{2\times10^{-2}\sqrt{2.25}}\sqrt{\frac{1^2}{3^2} + \frac{1^2}{1.25^2}} = \mathbf{8.67\,GHz} = \boldsymbol{f_{c_{11}}}$$

Problem 1: A plane wave is normally incident on a planar ineterface between air and muscle. The conductivity and dielectric constants of muscle are $\sigma_m = 0.889\, S/m$ and $\varepsilon_r = 71.7$. The intrinsic impedance of a conducting (or lossy) medium such as muscle or fat is given by:

$$\eta_c = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} e^{j\frac{1}{2}\tan^{-1}\left[\frac{\sigma}{\omega\varepsilon}\right]}$$

Determine the percentage of incident power absorbed by the muscle tissue at:

a) 100 Mhz

$$\eta_c = 28.5\, e^{j33.01^o} = 23.9 + j\,15.5$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{23.9 + j\,15.5 - 377}{23.9 + j\,15.5 + 377} = \frac{353\, e^{j177^o}}{401\, e^{j2.22^o}} = 0.878\, e^{j175^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1-\rho^2) = (1-0.878^2) = 0.229 = \mathbf{22.9\,\%}$$

b) 300 Mhz

$$\eta_c = 40.0\, e^{j18.4^o} = 37.9 + j\,12.6$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{37.9 + j\,12.6 - 377}{37.9 + j\,12.6 + 377} = \frac{339\, e^{j178^o}}{415\, e^{j1.74^o}} = 0.817\, e^{j176^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1-\rho^2) = (1-0.817^2) = 0.332 = \mathbf{33.2\,\%}$$

c) 915 Mhz

$$\eta_c = 44.1\, e^{j6.9^o} = 43.7 + j\,5.29$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{43.7 + j\,5.29 - 377}{43.7 + j\,5.29 + 377} = \frac{333\, e^{j179^o}}{420\, e^{j0.72^o}} = 0.792\, e^{j178^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1-\rho^2) = (1-0.792^2) = 0.373 = \mathbf{37.3\,\%}$$

d) 2.45 Ghz.

$$\eta_c = 44.6\, e^{j2.62^o} = 44.6 + j\,2.04$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.6 + j\,2.04 - 377}{44.6 + j\,2.04 + 377} = \frac{332\, e^{j180^o}}{421\, e^{j0.28^o}} = 0.789\, e^{j180^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1-\rho^2) = (1-0.789^2) = 0.378 = \mathbf{37.8\,\%}$$

Problem 1 Continued

Repeat (a) -- (d) if muscle is replaced by Fat $(\sigma_f = 0.155\, S/m \text{ and } \varepsilon_r = 71.7)$

a) 100 Mhz

$$\eta_f = 43.1\, e^{j10.6^o} = 42.4 + j\,8.01$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{42.4 + j\,8.01 - 377}{42.4 + j\,8.01 + 377} = \frac{335\, e^{j179^o}}{419\, e^{j1.09^o}} = 0.798\, e^{j178^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1 - \rho^2) = (1 - 0.798^2) = 0.363 = \mathbf{36.6\,\%}$$

b) 300 Mhz

$$\eta_f = 44.5\, e^{j3.72^o} = 44.4 + j\,2.89$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.4 + j\,2.89 - 377}{44.4 + j\,2.89 + 377} = \frac{333\, e^{j179.5^o}}{421\, e^{j0.39^o}} = 0.789\, e^{j179^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1 - \rho^2) = (1 - 0.789^2) = 0.377 = \mathbf{37.7\,\%}$$

c) 915 Mhz

$$\eta_f = 44.7\, e^{j1.23^o} = 44.7 + j\,0.96$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.7 + j\,0.96 - 377}{44.7 + j\,0.96 + 377} = \frac{332\, e^{j180^o}}{422\, e^{j0.13^o}} = 0.788\, e^{j180^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1 - \rho^2) = (1 - 0.788^2) = 0.379 = \mathbf{37.9\,\%}$$

d) 2.45 Ghz.

$$\eta_f = 44.7\, e^{j0.46^o} = 44.7 + j\,0.36$$

$$\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{44.7 + j\,0.36 - 377}{44.7 + j\,0.26 + 377} = \frac{332\, e^{j180^o}}{422\, e^{j0.05^o}} = 0.788\, e^{j180^o}$$

$$\frac{|(P_{av})_t|}{|(P_{av})_i|} = (1 - \rho^2) = (1 - 0.788^2) = 0.379 = \mathbf{37.9\,\%}$$

Problem 2: The electric field associated with a uniform plane wave propagating in air is given by: $E_i = 1000\cos(10^8 \pi t - \beta_0 z)\, a_x\, V/m$ The wave is normally incident on a dielectric medium $(\sigma_d = 0, \text{and}\, \varepsilon_d = 5\varepsilon_{0,}\, \mu_d = \mu_0)$

Determine the following:

(a) β_0 in air and β_d in dielectric material.

$$\beta_0 = \frac{\omega}{c} = \frac{10^8 \pi}{3 \times 10^8} = \frac{\pi}{3} = \mathbf{1.047\, rad/m} = \beta_{\mathbf{0}}$$

$$\beta_d = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{10^8 \pi}{3 \times 10^8}\sqrt{\frac{1}{5}} = \mathbf{2.342\, rad/m} = \beta_{\boldsymbol{d}}$$

(b) Reflection and transmission coefficients

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_0 \sqrt{\frac{1}{5}}, \eta_1 = \eta_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{1}{5}} - 1}{\sqrt{\frac{1}{5}} + 1} = -\mathbf{0.382} = \Gamma$$

$$\tau = 1 + \Gamma = 1 - 0.382 = \mathbf{0.618} = \tau$$

(c) Amplitudes of the reflected and transmitted E and H fields.

$$|E_{ot}| = |\tau| * E_0 = 0.618 * 1000 = \mathbf{618} = |\boldsymbol{E_{ot}}|, |H_{ot}| = \frac{E_{ot}}{\eta_2} = \frac{618}{120\pi\sqrt{\frac{1}{5}}} = \mathbf{3.67} = |\boldsymbol{H_{ot}}|$$

$$|E_{\text{or}}| = |\Gamma| * E_0 = 0.382 * 1000 = \mathbf{382} = |\boldsymbol{E}_{\mathbf{or}}|, |H_{\text{or}}| = \frac{E_{\text{or}}}{\eta_1} = \frac{318}{120\pi} = \mathbf{1.01} = |\boldsymbol{H}_{\mathbf{or}}|$$

(d) Reflected and transmitted power

$$P_{ave_t} = \frac{E_{ot}^2}{2\eta_2} = \frac{618^2}{240\pi\sqrt{\frac{1}{5}}} = \mathbf{1.13\, kW/m^2} = \boldsymbol{P_{ave_t}}$$

$$P_{ave_r} = \frac{E_{\text{or}}^2}{2\eta_1} = \frac{318^2}{240\pi} = \mathbf{194\, W/m^2} = \boldsymbol{P_{ave_r}}$$