Practice Problem 12.1: Consider the waveguide of Example 12.1.

Calculate the phase constant, phase velocity and wave impedance for TE_{10} and TM_{11} modes at the operating frequency of 15 Ghz.

A) TE_{10}

$$\sqrt{1-\left(\frac{f_c}{f}\right)^2}=\sqrt{1-\left(\frac{3}{15}\right)^2}=\sqrt{0.96}=0.9798, \beta=\frac{\omega}{u_0}=2\pi f/c$$

$$\beta=\beta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}=\frac{2\pi\, 15\times10^9}{3\times10^8}(0.9798)=\mathbf{615.6\, rad/s}=\beta$$

$$u=\frac{\omega}{\beta}=\frac{2\pi\, 15\times10^9}{615.6}=\mathbf{153\times10^6\, m/s=u}$$

$$\eta'=\sqrt{\frac{\mu}{\varepsilon}}=60\pi, \eta_{TE}=\frac{60\pi}{\sqrt{0.9798}}=\mathbf{192.4}\,\Omega=\eta_{\boldsymbol{TE}}$$

B) $TM_{11}, f_c=3\,GHz*\sqrt{7.25}=8.08\,GHz$

$$\sqrt{1-\left(\frac{f_c}{f}\right)^2}=\sqrt{1-\left(\frac{8.08}{15}\right)^2}=0.8425, \beta=\frac{\omega}{u_0}=4\pi f/c$$

$$\beta=\beta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}=\frac{4\pi\, 15\times10^9}{3\times10^8}(0.8425)=\mathbf{529.4\, rad/s}=\beta$$

$$u=\frac{\omega}{\beta}=\frac{2\pi\, 15\times10^9}{529.3}=\mathbf{178\times10^6\, m/s=u}$$

$$\eta'=\sqrt{\frac{\mu}{\varepsilon}}=60\pi, \eta_{TE}=60\pi\sqrt{0.8425}=\mathbf{158.8}\,\Omega=\eta_{\boldsymbol{TE}}$$

Practice Problem 12.2: An air-filled 5- by 2-cm waveguide has

$$E_{zs}\, 20\sin 40\pi x \sin 50\pi y\, e^{-j\beta z}\, V/m$$

at 15GHz.

A) What mode is being propagated?

Since $E_z \neq 0$, this is a TM mode

$$E_z=E_0\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

$$E_0=20, \frac{m\pi}{a}=40\pi=\frac{m\pi}{0.05\, meters}\rightarrow \boldsymbol{m=2}, \frac{n\pi}{b}=50\pi=\frac{n\pi}{0.02\, meters}\rightarrow \boldsymbol{n=1}$$

$$\boldsymbol{TM\, mode=TM_{21}}$$

Practice Problem 12.2 Continued:

B) Find β

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{40^2+50^2} = 9.6\,GHz = f_c$$

$$\beta = \frac{\omega}{u'}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi f}{c}\sqrt{f^2 - f_c^2} = \frac{2\pi\times10^9}{3\times10^8}\sqrt{225-92.25}$$

$$\beta = \mathbf{241.3\,rad/m}$$

C) Determine E_y/E_z

$$E_{xs} = \frac{-j\beta}{h^2}(40\pi)\,20\cos(40\pi x)\sin(50\pi y)e^{-j\beta z}$$

$$E_{ys} = \frac{-j\beta}{h^2}(50\pi)\,20\sin(40\pi x)\cos(50\pi y)e^{-j\beta z}$$

$$\frac{E_y}{E_x} = \frac{\frac{-j\beta}{h^2}(50\pi)\,20\cos(40\pi x)\sin(50\pi y)e^{-j\beta z}}{\frac{-j\beta}{h^2}(40\pi)\,20\sin(40\pi x)\cos(50\pi y)e^{-j\beta z}}$$

$$\frac{\mathbf{E_y}}{\mathbf{E_x}} = \mathbf{1.25\tan(40\pi x)\cot(50\pi y)}$$

Problem 12.3: A 1-cm X 2-cm waveguide is filled with deionized water with $\varepsilon_r = 81$. If the operating frequency is 4.5GHz, determine:

A) all possible propagating modes

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2*9*10^{-2}}\sqrt{\left(\frac{m}{2}\right)^2 + \left(\frac{n}{1}\right)^2} = \frac{15}{18}(\sqrt{m^2+4n^2})\times10^9$$

$$\mathbf{f_c = \frac{5}{6}(\sqrt{m^2+4n^2})\times10^9\,Hz}$$

Frequency (Ghz)	***N = 0***	***N = 1***	***N = 2***
M = 0	DNE	1.667	3.333
M = 1	0.833	1.863	3.436
M = 2	1.667	2.357	3.727
M = 3	2.500	3.004	4.167
M = 4	3.333	3.727	Too high
M = 5	4.167	4.488	Too high

Problem 12.3 Continued:

B) The intrinsic impedance of the highest mode

The highest possible mode is TE_{51} or TM_{51}.

$$\eta' = 120\frac{\pi}{9} = 41.89, \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{4.488}{4.5}\right)^2} = 0.073$$

$$\eta_{TE_{51}} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{41.89}{0.073} = \mathbf{573.8}\,\Omega = \eta_{\boldsymbol{TE_{51}}}$$

$$\eta_{TM_{51}} = \eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 41.89*0.073 = \mathbf{3.058}\,\Omega = \eta_{\boldsymbol{TM_{51}}}$$

C) The group velocity of the lowest mode

The lowest possible mode is TE_{10}.

$$u' = \frac{c}{9,}\; u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{3\times10^8}{9}\sqrt{1-\left(\frac{0.833}{4.5}\right)^2} = \mathbf{32.76\times10^6\, m/s = u_g}$$

Problem 12.5: A tunnel is modeled as an air-filled metallic rectangular waveguide with dimensions *a* = 8m and *b* = 16m. Determine whether the tunnel will pass:

For the dominant mode TE_{01} , $f_c = \frac{c}{2b} = \frac{3\times10^8}{2*16} = 9.375\,MHz$

A) a 1.5-MHz AM broadcast signal

The AM signal will not pass because it is below the cutoff frequency

B) a 120-MHz FM broadcast signal

The FM signal will pass because it is above the cutoff frequency

Problem 12.9: An air-filled rectangular waveguide had cross-sectional dimensions *a* = 6cm and *b* = 3cm. Given that

$$E_z = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12}t - \beta z)\, V/m$$

Calculate the intrinsic impedance of this mode and the average power flow in the guide.

Since $E_z \neq 0$ this must be TM_{23} mode (m=2, n=3).

Since a = 2b,

$$f_c = \frac{c}{4b}\sqrt{m^2 + 4n^2} = \frac{3\times 10^8}{4*3*10^{-2}}\sqrt{4+36} = 15.81\, GHz$$

$$f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.2\, GHz$$

$$\eta_{TM} = 120\pi\sqrt{1-\left(\frac{15.81}{159.2}\right)^2} = -\mathbf{375.1}\,\Omega = \eta_{TM}$$

$$Poynt_{ave} = \frac{|E_{xs}{}^2| + |E_{ys}{}^2|}{2\eta_{TM}}\, a_z$$

$$Poynt_{ave} = \frac{\beta^2 E_0^2}{2h^4\eta_{TM}}\left[\left(\frac{2\pi}{a}\right)^2\cos^2\left(\frac{2\pi x}{a}\right)\sin^2\left(\frac{3\pi y}{b}\right) + \left(\frac{3\pi}{b}\right)^2\cos^2\left(\frac{2\pi x}{a}\right)\sin^2\left(\frac{3\pi y}{b}\right)\right] a_z$$

$$P_{ave} = \int Poynt_{ave}\, dS = \int_{x=0}^{a}\int_{y=0}^{b} Poynt_{ave}\, dxdy = \frac{\beta^2 E_0^2}{2h^4\eta_{TM}}\frac{1}{4}\left[\frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2}\right] = \frac{\beta^2 E_0^2}{8h^2\eta_{TM}}$$

$$\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{f}{f_c}\right)^2} = \frac{10^{12}}{3\times 10^8}\sqrt{1-\left(\frac{15.81}{159.2}\right)^2} = 3.317\times 10^3$$

$$h^2 = \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} = \frac{10\pi^2}{b^2} = 1.098\times 10^5$$

$$P_{ave} = \frac{(3.317\times 10^3)^2(25)}{(8)(1.098\times 10^5)(375.4)} = \mathbf{0.8347\, W} = \boldsymbol{P_{ave}}$$

Problem 12.10: In an air-filled rectangular waveguide, a TE mode operating at 6GHz has

$$E_y = 5\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\cos(\omega t - 12z)\,V/m$$

Determine:

A) the mode of operation

Since m = 2 and n = 1, we have $\boldsymbol{TE_{21}}$.

B) the cutoff frequency

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$\beta c = \sqrt{\omega^2 - \omega_c^2} \rightarrow \omega_c^2 = \sqrt{\omega^2 - \beta^2 c^2}$$

$$f_c = \frac{\omega_c}{2\pi} = \sqrt{f^2 - \left(\frac{\beta c}{2\pi}\right)^2} = \sqrt{(6\times10^9)^2 - \left(\frac{12*3\times10^8}{2\pi}\right)^2} = \mathbf{5.973\,GHz} = \boldsymbol{f_c}$$

C) the intrinsic impedance

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{5.973}{6}\right)^2}} = \mathbf{3978}\,\Omega = \boldsymbol{\eta_{TE}}$$

D) H_x

For TE mode

$$\mathrm{E_y} = \frac{\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$\mathrm{H_x} = \frac{-\beta}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin(\omega t - \beta z)$$

$$\beta = 12, m = 2, n = 1$$

$$E_{0y} = \frac{\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_{0,}\; H_{0x} = \frac{\beta}{h^2}\left(\frac{m\pi}{a}\right)H_0$$

$$\eta_{TE} = \frac{E_{0y}}{H_{0x}} = \frac{\omega\mu}{\beta} = \frac{2\pi*6\times10^9*4\pi*10^{-7}}{12} = 4\pi^2\times10^2 = 3.948\times10^3$$

$$H_{0x} = \frac{E_{0y}}{\eta_{TE}} = \frac{5}{4\pi^2\times10^2} = 1.267\,\mathrm{mA/m}$$

$$\mathbf{H_x = 1.267\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin(\omega t - \beta z)\,mA/m}$$

Problem 12.11: In an air-filled rectangular waveguide with *a* = 2.286cm and *b* = 1.016cm, the *y*-component of the TE mode is given by

$$E_y = \sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10\pi\times 10^{10}t - \beta z)\, V/m$$

Find:

A) the operating mode

Since m = 2 and n = 3, the **mode is** $\boldsymbol{TE_{23}}$**.**

B) the propaation constant γ

$$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi f}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$But, f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times 10^8}{2\times 10^{-2}}\sqrt{\left(\frac{2}{2.286}\right)^2+\left(\frac{3}{1.016}\right)^2} = 46.19\,GHz, f = 50\,GHz$$

$$\beta = \frac{2\pi * 50\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{46.19}{50}\right)^2} = 400.68\,rad/m$$

$$\gamma = \boldsymbol{j}\beta = \boldsymbol{j400.7/m}$$

C) the intrinsic impedance η

$$\eta = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{46.19}{50}\right)^2}} = \mathbf{985.3}\,\Omega = \eta$$