Problem 12.19: For TE_{01} mode,

$$E_{xs}=\frac{j\omega\mu\pi}{bh^2}H_0\sin\left(\frac{\pi y}{b}\right)e^{-\gamma z},E_{ys}=0$$

Find $Poynt_{ave}$ and P_{ave}

$$Poynt_{ave}=\frac{|E_{xs}|^2+|E_{ys}|^2}{2\eta}\hat{a}_z=\frac{\omega^2\mu^2\pi^2}{2\eta b^2h^4}H_0^2\sin^2\left(\frac{\pi y}{b}\right)\hat{a}_z$$

$$P_{ave}=\int Poynt_{ave}dS=\frac{\omega^2\mu^2\pi^2}{2\eta b^2h^4}H_0\int_{x=0}^{a}\int_{y=0}^{b}\sin^2\left(\frac{\pi y}{b}\right)dx\,dy$$

$$P_{ave}=\frac{\omega^2\mu^2\pi^2}{2\eta b^2h^4}H_0^2\frac{ab}{2}$$

$$But,h^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2=\frac{\pi^2}{b^2},so...$$

$$P_{ave}=\frac{\omega^2\mu^2ab^3H_0^2}{4\pi^2\eta}$$

Problem 12.20: A 1-cm x 2-cm waveguide is made of copper $(\sigma_c=5.8\times10^7 S/m)$ and filled with a dielectric material for which $\varepsilon=2.6\varepsilon_0,\mu=\mu_0$ and $\sigma_d=10^{-4}S/m.$ If the guide operates at 9 Ghz, evaluate α_c and α_d for:

$$R_s=\sqrt{\frac{\pi\mu f}{\sigma_c}}=\sqrt{\frac{\pi(9\times10^9)(4\pi\times10^{-7})}{5.8\times10^7}}=24.75\,m\Omega$$

$$f_{c10}=\frac{u'}{2a}=\frac{c}{2a\sqrt{\varepsilon_r}}=\frac{3\times10^8}{2(0.02)\sqrt{2.6}}=4.65\,GHz$$

$$f_{c11}=\frac{u'}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}=\frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}=\frac{3\times10^8}{2\sqrt{2.6}}\sqrt{\frac{1}{0.02^2}+\frac{1}{0.01^2}}=10.40\,GHz$$

A) TE_{10}

$$\alpha_c=\frac{2R_s}{b\eta'\sqrt{1-\left[\frac{f_c}{f}\right]^2}}\left(\frac{1}{2}+\frac{b}{a}\left[\frac{f_c}{f}\right]^2\right)=\frac{2*0.02475}{0.01*\frac{120\pi}{\sqrt{2.6}}\sqrt{\frac{4.65^2}{9^2}}}\left(\frac{1}{2}+\frac{1}{2}\left[\frac{4.65}{9}\right]^2\right)$$

$$\boldsymbol{\alpha_c=25.96\times10^{-3}Np/m}$$

Problem 12.20 continued)

$$\alpha_d=\frac{\sigma_d\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}=\frac{10^{-4}\frac{120\pi}{\sqrt{2.6}}}{2\sqrt{1-\left(\frac{4.65}{9}\right)^2}}=\mathbf{0.01365\,Np/m}=\alpha_{\mathbf{d}}$$

B) TM_{11}

Since $f_{c11}=10.4\,GHz>9\,GHz=f$, **the signal is below the cutoff frequency and is not passed, there is no attenuation.**

Problem 12.21: Consider the waveguide of Example 12.1. A 4cm-square waveguide is filled with a dielectric with complex permittivity $\varepsilon_c=16\varepsilon_0(1-j10^{-4})$ and is excited with the TM_{21} mode. If the waveguide operates at 10% above the cutoff frequency, calculate attenuation α_d. How far can the wave travel down the guide before its magnitude is reduced by 20%?

$$\varepsilon_c=\varepsilon'-j\varepsilon''=\varepsilon-j\frac{\sigma}{\omega}=16\varepsilon_0(1-j10^{-4})=16\varepsilon_0-j16\varepsilon_0\times10^{-4}$$

$$\text{Thus},\ \varepsilon=16\varepsilon_0\ \text{and}\ \frac{\sigma}{\omega}=16\varepsilon_0\times10^{-4}$$

For TM_{21} mode

$$f_c=\frac{u'}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}=\frac{3\times10^8}{2\sqrt{16}}\sqrt{\frac{2^2}{0.04^2}+\frac{1^2}{0.04^2}}=2.096\,GHz$$

$$f=1.1f_c=2.306\,GHz$$

$$\sigma=\omega16\varepsilon_0\times10^{-4}=2\pi(2.306\times10^9)(16)(8.854\times10^{-12})(10^{-4})=205\times10^{-6}$$

$$\eta'=120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}=\frac{120\pi}{\sqrt{16}}=30\pi$$

$$\alpha_d=\frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}=\frac{(205\times10^{-6})(30\pi)}{2\sqrt{1-\left(\frac{2.096}{2.306}\right)^2}}=\mathbf{0.0232\,Np/m}=\alpha_{\mathbf{d}}$$

We want to find the distance that the magnitude is $0.8E_{0,}$

$$0.8E_0=E_0e^{-\alpha_d z}\rightarrow z=\frac{1}{\alpha_d}\ln\frac{1}{0.8}=\frac{1}{0.0232}\ln\frac{1}{0.8}=\mathbf{9.6\,m}=\mathbf{z}$$

Problem 12.29: In a rectagular resonant cavity, which mode is dominant when:

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

For TM mode to z, $m = 1,2,3,\ldots\ n = 1,2,3,\ldots\ p = 0,1,2,\ldots$

For TE mode to z, $m = 0,1,2,\ldots\ n = 0,1,2,\ldots\ p = 1,2,3,\ldots (\text{Note}: m = n \neq 0)$

A) a < b < c

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{011} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{b^2} + \frac{1}{c^2}} < \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Thus the dominant mode is $\mathbf{TE_{011}}$

B) a > b > c

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{c^2}} > \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Thus the dominant mode is $\mathbf{TM_{110}}$

C) a = c > b

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{c^2}} < \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Thus the dominant mode is $\mathbf{TE_{101}}$

Problem 12.32: An air-filled cubical cavity operates at a resonant frequency of 2 GHz when excited at the TE_{101} mode. Determine the dimensions of the cavity.

$$f_{r101} = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}; \text{for a cubical cavity}, a = b = c.$$

$$\text{Thus}, f_{r101} = \frac{u'}{2}\sqrt{\frac{2}{a^2}} = \frac{u'}{2a}\sqrt{2} \rightarrow a = \frac{u'}{f_{r101}\sqrt{2}} = \frac{3\times10^8}{2\times10^9\sqrt{2}}$$

$$\mathbf{a = 10.61\,cm = b = c}$$