Problem 13.3: A 2-A source operating at 300MHz feeds a Hertzian dipole of length 5 mm situated at the origin. Find E_s and H_s at $(10, 30^o, 90^o)$

First we need to determine what field we are in (near, far, etc)
For far field, $\beta r \gg 1$ or $2\pi r \gg \lambda$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1\,m,\ \beta = \frac{2\pi}{\lambda} = 2\pi\,rad/s,\ r = 10,\ \vartheta = 30^o,\ \phi = 90^o$$

$$\textit{Since}\ 20\pi \gg 1, \text{we are in far field}$$

$$H_{\phi s} = \frac{j I_0 \beta\, dl}{4\pi r}\sin\vartheta\, e^{-j\beta r},\ E_{\vartheta s} = \eta H_{\phi s},\ H_{rs} = H_{\vartheta s} = E_{rs} = E_{\phi s} = 0$$

$$H_{\phi s} = \frac{j(2)(2\pi)(5\times10^{-3})}{4\pi(10)}\sin(30^o)\, e^{-j(2\pi)(10)} = \boldsymbol{j250\,\mu A/m}$$

$$\eta = \eta_0 = 120\pi$$

$$E_{\vartheta s} = \eta H_{\phi s} = (120\pi)(j250\times10^{-6}) = \boldsymbol{j94.25\,mV/m}$$

Problem 13.4:

A) Instead of a constant current distribution assumed for the short dipole of Section 13.2, assume a triangular current distribution $I_S = I_0\left(1 - \frac{2|z|}{l}\right)$ shown in Fig 13.23. Show that $R_{rad} = 20\pi^2\left[\frac{l}{\lambda}\right]^2$, which is one-fourth of that in eq/ (13.13). Thus R_{rad} epends on the current distribution.

$$A_{zs} = \frac{e^{-j\beta r}}{4\pi r}\int_{-0.5}^{0.5} I_0\left(1 - \frac{2|z|}{l}\right) e^{j\beta\cos\vartheta}\,\partial z$$

$$= \frac{e^{-j\beta r}}{4\pi r} I_0\left[\int_{-0.5}^{0.5}\left(1 - \frac{2|z|}{l}\right) e^{j\beta\cos\vartheta}\,dz + j\int_{-0.5}^{0.5}\left(1 - \frac{2|z|}{l}\right)\sin(\beta z\cos\vartheta)\,dz\right]$$

$$= \frac{e^{-j\beta r}}{4\pi r} 2 I_0 \int_{-0.5}^{0.5}\left(1 - \frac{2|z|}{l}\right)\sin(\beta z\cos\vartheta)\,dz$$

$$= \frac{I_0 e^{-j\beta r}}{2\pi r \beta^2 \cos^2\vartheta}\left(\frac{2}{l}\right)\left[l - \cos\left(\frac{\beta l}{2}\cos\vartheta\right)\right]$$

$$E_s = -j\omega\mu A_s \rightarrow E_{\vartheta s} = j\omega\mu\sin\vartheta\, A_{zs} = j\beta\eta\sin\vartheta\, A_{zs}$$

$$E_{\vartheta s} = \frac{\eta I_0 e^{-j\beta r}}{\pi r l} * \frac{\sin\vartheta\left[l - \cos\left(\frac{\beta l}{2}\cos\vartheta\right)\right]}{\beta\cos^2\vartheta}$$

Problem 13.4 Continued:

$$If\ \frac{\beta l}{2} \ll 1, \cos\left(\frac{\beta l}{2}\cos\vartheta\right) = l - \frac{\left(\frac{\beta l}{2}\cos\vartheta\right)^2}{2!}$$

$$E_{\phi s} = \frac{\eta I_0 e^{-j\beta r}}{\pi r l} * \frac{\sin\vartheta\left[l - l + \frac{\left(\frac{\beta l}{2}\cos\vartheta\right)^2}{2!}\right]}{\beta\cos^2\vartheta} = \frac{j\eta I_0}{8\pi r}\beta l e^{-j\beta r}\sin\vartheta, H_{\phi s} = \eta E_{\vartheta s}$$

$$P_{ave} = \frac{|E_{\phi s}|^2}{2\eta}, P_{rad} = \oint P_{ave}\, dS$$

$$P_{rad} = \int_0^{\pi}\int_0^{2\pi} \frac{n}{2}\left(\frac{I_0\beta l}{8\pi}\right)^2 \frac{1}{r^2}\sin^2\vartheta\, r^2 \sin\vartheta\, d\phi\, d\vartheta$$

$$= \frac{n}{2}\left(\frac{I_0\beta l}{8\pi}\right)^2 \frac{1}{r^2}(r^2)\int_0^{\pi}\int_0^{2\pi} \sin^3\vartheta\, d\phi\, d\vartheta = 10\pi^2 I_o^2\left(\frac{l}{\lambda}\right)^2$$

$$\boldsymbol{P_{rad} = \frac{1}{2} I_0^2 R_{rad} \text{ or } R_{rad} = 20\pi^2\left(\frac{l}{\lambda}\right)^2}$$

B) Calculate the length of the diploe that will result in a radiation resistance of 0.5Ω.

$$0.5 = 20\pi^2\left(\frac{l}{\lambda}\right)^2 \rightarrow \boldsymbol{l \simeq 0.05}\lambda$$

Problem 13.7: A 1m-long car radio antenna operates in the AM frequency of 1.5MHz. How much current is required to transmit 4 W of power?

This is a monopole antenna.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.5\times10^6} = 200$$

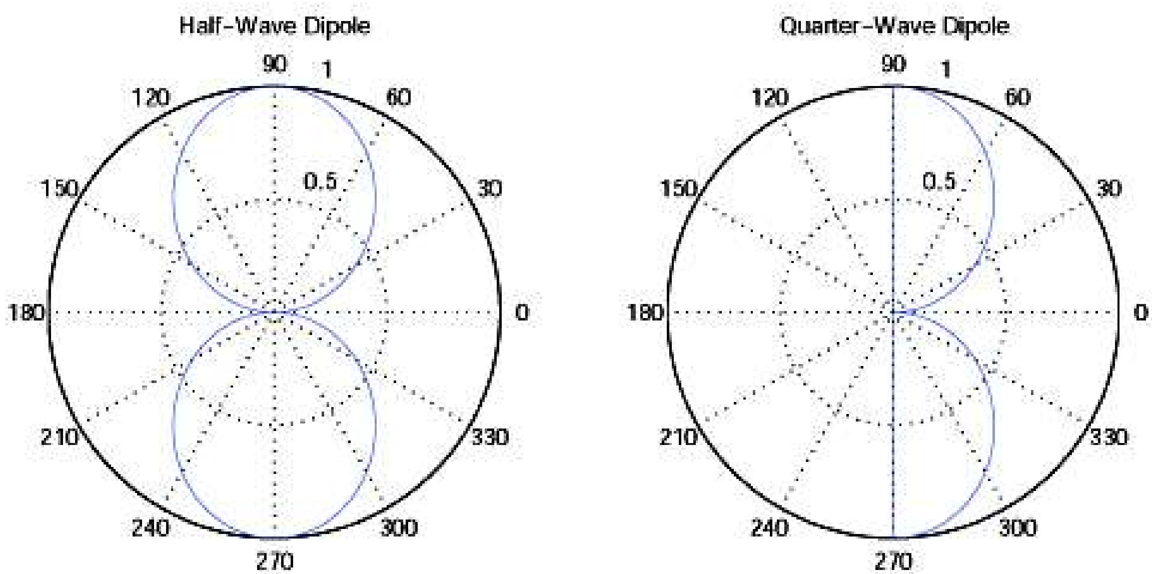
$$Since\,,\, \lambda \gg l\,,\, it\, is\, a\, Hertian\, monopole$$

$$R_{rad} = \frac{1}{2} 80\,\pi^2 \left(\frac{dl}{\lambda}\right)^2 = 40\,\pi^2 \left(\frac{1}{200}\right)^2 = 9.87\, m\,\Omega$$

$$P_{rad} = P_t = \frac{1}{I_0^2} R_{rad} \rightarrow I_0 = \sqrt{\frac{2\,P_t}{R_{rad}}} = \sqrt{\frac{2(4)}{9.87\times10^{-3}}} = \mathbf{28.47\ A}$$

Problem 13.12 : Sketch the normalized E-field and H-field patterms for

$$f(\vartheta) = \left| \frac{\cos\left(\frac{\pi}{2}\cos\vartheta\right)}{\sin\vartheta} \right|$$



Problem 13.14 : In free space, an antenna has a far-zone field given by:

$$E_s = \frac{5\sin 2\vartheta}{r} e^{j\beta r} \hat{a}_\theta V/m$$

where $\beta = \omega\sqrt{\mu_0 \varepsilon_0}$. Determine the radiated power.

$$P_{ave} = \frac{|E_s|^2}{2\eta}\hat{a}_r = \frac{25\sin^2 2\vartheta}{2\eta r^2}\hat{a}_r$$

$$P_{rad} = \oint P_{ave} dS = \frac{25}{2\eta}\int_0^\pi \int_0^{2\pi} (2\sin\vartheta\cos\vartheta)^2 \sin\vartheta \, d\phi \, d\vartheta$$

$$= \frac{20(2\pi)}{2(120\pi)}\int_0^\pi 4\sin^2\vartheta\cos^2\vartheta \, d(-\cos\vartheta)$$

$$= \frac{5}{6}\int_0^\pi (\cos^4\vartheta - \cos^2\vartheta) \, d(-\cos\vartheta)$$

$$= \frac{5}{6}\left[\frac{\cos^5\vartheta}{5} - \frac{\cos^3\vartheta}{3}\right]_0^\pi = \frac{5}{6}\left(-\frac{2}{5}+\frac{2}{3}\right) = \mathbf{222.2\, mW}$$

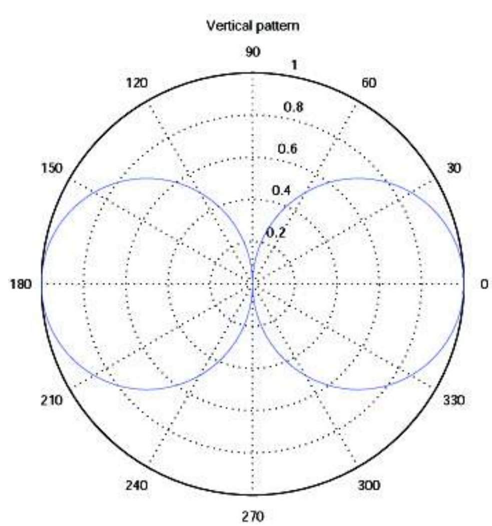
Problem 13.15 : At the far field, the electric field produced by an antenna is:

$$E_s = \frac{10}{r} e^{j\beta r}\cos\vartheta\cos\phi\,\hat{a}_z$$

Sketch the vertical pattern of the antenna. Your plot should include as many points as possible.

$f(\vartheta) = |\cos\vartheta\cos\phi|$

For the vertical pattern, $\phi = 0 \rightarrow f(\vartheta) = |\cos\vartheta|$.



Problem 13.17 : At the far field, an antenna produces

$$P_{ave} = \frac{2\sin\vartheta\cos\phi}{r^2}\hat{a}_r W/m^2, 0<\vartheta<\pi, 0<\phi<\frac{\pi}{2}$$

Calculate the directive gain and the directivity of the antenna.

$$G_D = \frac{U}{U_{ave}} = \frac{2\pi r^2 P_{Ave}}{\oint P_{ave}\partial S} = \frac{8\pi\sin\vartheta\cos\phi}{\oint P_{ave}\partial S}$$

$$However, \oint P_{ave}\partial S = \int_0^{\pi}\int_0^{\frac{\pi}{2}} 2\sin\vartheta\cos\phi\sin\vartheta\, d\phi\, d\vartheta$$

$$= 2\int_0^{\frac{\pi}{2}}\cos\phi\, d\phi \int_0^{\pi}\sin^2\vartheta\, d\vartheta = 2(1)\left(\frac{\pi}{2}\right) = \pi$$

$$Thus, G_d = \frac{8\pi\sin\vartheta\cos\phi}{\pi} = \mathbf{8\sin\vartheta\cos\phi = G_d}$$