• Answer all questions?
• Closed books
• Closed notes except for equation sheet.
• No crib notes
• No headphons, cell phones, or pagers.
• No hats
• No guests or vistors during exam.
• See attached equation sheets.

Signature:______________________________
Problem 1: (28 points)

The magnetic field intensity of a plane wave of angular frequency \( \omega = 10^5 \text{ rad/sec} \) is given in phasor form as:

\[ H_s = 0.1 e^{0.1y} e^{j2y} a_x \text{ A/m} \]

(a) The wavelength is: \( \beta = 2, \lambda = 2\pi/\beta = \pi \text{ meters} \)

(b) The phase velocity is: \( u = \omega/\beta = 10^5/2 = 5 \times 10^4 \text{ m/s} \)

(c) The direction of propagation is: \(-a_y\)

(d) The direction of the electric field intensity vector for this field is: \(-a_z\)

(e) The expression of the actual magnetic field intensity is:

\[ H(y,t) = 0.1 e^{0.1y} \cos(\omega t + 2y) a_x \text{ A/m} \]

(f) The amplitude of the magnetic field after propagating a distance 100 meters into the medium would be:

\[ \text{e}^{-10} \text{ times its initial amplitude.} \]
Problem 2: (24 points)

A perpendicularly polarized plane wave is incident on a boundary between two materials with parameters given in the figure below:

\[ \begin{align*}
(1) & \quad \varepsilon = 9\varepsilon_0, \mu = \mu_0, \\
& \quad \sigma = 0 \\
(2) & \quad \varepsilon = \varepsilon_0, \mu = \mu_0, \\
& \quad \sigma = 0
\end{align*} \]

a) If \( \theta_1 = 15^\circ \), the angle \( \theta_2 = _______75^\circ_______ \)

b) If \( \theta_1 = 15^\circ \), the angle \( \theta_3 = _____51^\circ_____ \)

\( n_1 \sin \theta_1 = n_2 \sin \theta_3 \) and \( n_1 = c/u = 3 \), \( n_2 = 1 \)

c) If the reflected field is zero, the angle \( \theta_1 = _____45^\circ_____ \)

\[ \begin{align*}
E_r = \Gamma_T E_i, \quad \Gamma_T = 0, \quad \theta_1 = \theta_{BT} \\
\tan \theta_{BT} = \sqrt{\frac{\mu_2}{\mu_1}} = 1
\end{align*} \]

d) If \( \theta_1 = 0 \), determine the ratio of the amplitude of the reflected electric field intensity to the amplitude of the incident electric field intensity.

\[ \frac{E_r}{E_i} = \Gamma, \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 40\pi}{120\pi + 40\pi} = 0.5 \]
Problem 3: (16)

A plane wave in air has:

$$E_i = 100a_y \sin(wt - x - \sqrt{3}z)$$

is incident on a perfect conductor slab in region $z > 0$. Determine the following:

(a) Propagation vector of the incident $E_i$ field? Show $E_i$ on the above plot? (4)

$$k_i = a_x + a_z \sqrt{3}$$

(b) Propagation constant of the reflected $E_r$ field? Sketch on the above plot? (4)

$$k_r = a_x - a_z \sqrt{3}$$

(c) Angle of reflection? (4) \[
\tan \theta_r = \frac{k_x}{k_y} = \frac{1}{\sqrt{3}} \\
\theta_r = 30^\circ
\]

(d) Transmitted Electric field $E_t$? (2)

Medium 2 is perfect conductor, \( E_t = 0 \)

(e) Reflection coefficient $\Gamma$ at interface $z=0$? (2)

$$\eta_2 = 0, \quad \Gamma = -1.$$
Problem 4: (20)

The above waveguide has perfect conducting metal walls and extends to $\infty$ in the z-diection. It has dimensions $a = 8$ cm and $b = 6$ cm (a along x-axis) and is filled with lossless dielectric ($\mu = \mu_0$ and $\varepsilon = 9\varepsilon_0$). One of the TE mode has the following component:

$$E_{ys} = \frac{j \omega \mu}{h^2} \left( \frac{2\pi}{a} \right) H_o \sin \left( \frac{2\pi x}{a} \right) \cos \left( \frac{3\pi y}{b} \right) e^{-\gamma z}$$

Determine the following:

a) The TE mode of operation $(m,n)$ is? (4)

$$m=2, \ n=3$$

b) The phase velocity $u'$ in the unbounded dielectric medium is: (4)

$$u' = 1/\sqrt{\mu\varepsilon} = 1/\sqrt{9\mu_0\varepsilon_0} = 10^8 \text{ m/s}$$

c) The cut off frequency for this mode is: (4) $f_c = 2.8$ GHz

d) The group velocity for a 5 GHz signal? (5)

$$U_g = 10^8 \sqrt{1 - (2.8/5)^2} \text{ m/s}$$

e) The electric field vector $\vec{E}$ for the TE$_{10}$ mode? (5)

$$E_{xs} = 0, \ E_{zs} = 0, \ \vec{E} = a_y E_{ys}$$

$$E_{ys} = \frac{j \omega \mu}{h^2} \left( \frac{\pi}{a} \right) H_o \sin \left( \frac{2\pi x}{a} \right) e^{-\gamma z}$$
Problem 5: (12)

In an air filled wave guide the TE\textsubscript{10} mode has cutoff frequency $f_c = 12$ GHz. The TE\textsubscript{01} mode has $f_c = 5$ GHz.

a) The dimensions of the guide are:

\[ a = \text{____} 1.25 \text{ cm} \text{____} \quad \text{and} \quad b = \text{____} 3 \text{ cm} \text{___________} \]

b) The cutoff frequency for the TM\textsubscript{20} mode when the waveguide is filled with a medium: \( \mu = \mu_o \), \( \varepsilon = 4\varepsilon_o \)

**TM\textsubscript{20} mode is not supported.** Cut frequency has no meaning