

October 21, 2005 Duration: 50 minutes	EE351 Midterm Exam. # 2 (Closed notes & book)	Fall 2005 School of EECS/ WSU
Name: _____-Master solution-_____		ID #: _____

- Answer all questions
- Closed books
- Closed notes (two pages of equations allowed)
- No crib notes
- No headphones, cell phones, or pagers.
- No hats
- No guests or visitors during exam.
- See attached equation sheets.

Signature: \_\_\_\_\_

	Possible Points	Grade point
1	25	
2	25	
3	25	
4	25	
	100	

Problem 1:

A rectangular copper waveguide filled with a dielectric ( $\epsilon = 4 \epsilon_0$ ) has dimensions  $a = 1.5$  cm, and  $b = 1$  cm. Determine the following for the  $TE_{10}$  mode at 10 GHz.

a) medium velocity  $u'$  (3):

$$u' = \frac{1}{\sqrt{4\mu_0\epsilon_0}} = \frac{c}{2} = 1.5 \times 10^8 \text{ m/s}$$

b)  $\beta_{10}$  (5):

$$f_{c10} = \frac{u'}{2a} = 5 \text{ GHz}$$

$$\beta' = \frac{\omega}{u'} = \frac{2\pi \times 10 \times 10^9}{1.5 \times 10^8} = 418.9 \text{ m}^{-1}$$

$$\beta_{10} = \beta' \sqrt{1 - (f_c / f)^2} = 362.8$$

c)  $\lambda_{10}$  (4)

$$\lambda_{10} = \frac{2\pi}{\beta_{10}} = 0.0173 \text{ m}$$

d)  $\eta_{10}$  (4):

$$\eta' = \sqrt{\frac{\mu_0}{4\epsilon_0}} = 377 / 2 = 188.5 \Omega$$

$$\eta_{10} = \frac{\eta'}{\sqrt{1 - (f_c / f)^2}} = 211.3 \Omega$$

e) Phase velocity  $u_p$  (5)

$$u_p = \frac{u'}{\sqrt{1 - (f_c / f)^2}} = 1.732 \times 10^8 \text{ m/s}$$

f) What is the value of  $\lambda_{10}$  if the frequency is changed to 4 GHz? (4)

Frequency  $f = 4$  GHz is less than cut-off frequency. No propagation. No  $\lambda$  is defined.

Problem 2:

A 5.0 cm by 3.0 cm rectangular waveguide is filled with a dielectric material ( $\mu = \mu_0$ ,  $\epsilon = 4\epsilon_0$ ,  $\sigma_d = 0$ ). Given:

$$E_z = 2 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(\pi 10^{11} t - \beta z) \text{ A/m}$$

Determine the following:

(a) Mode of operation :

TM<sub>13</sub> mode

(b) Cutoff frequency:

$$u' = \frac{1}{\sqrt{4\mu_0\epsilon_0}} = \frac{c}{2} = 1.5 \times 10^8 \text{ m/s}$$

$$f_c = \frac{u'}{2} \sqrt{\frac{1}{a^2} + \left(\frac{3}{b}\right)^2} = 7.65 \text{ GHz}$$

(c) Propagation constant  $\beta$

$$f_{c13} = 7.65 \text{ GHz}$$

$$\beta' = \frac{\omega}{u'} = \frac{2\pi \times 50 \times 10^9}{1.5 \times 10^8} = 2094.7418.9 \text{ m}^{-1}$$

$$\beta = \beta' \sqrt{1 - (f_c / f)^2} = 2070 \text{ m}^{-1}$$

(d) The group velocity for the (m = 1, n = 0) mode

TM<sub>10</sub> is not supported. Group velocity has no meaning

Problem 3:

Circle the correct answer(s): (show all calculations if needed)

(1) The dominant mode for a rectangular waveguide is: (4)

- (a)  $TE_{11}$       (b)  $TM_{11}$     (c)  $TE_{101}$     (d)  $TE_{10}$

(2) The  $TM_{10}$  mode can exist in a rectangular waveguide. (4)

- (a) True      (b) False

(3) For the  $TE_{30}$  mode which of the following field components exist (i.e. non-zero): (4)

- (a)  $E_x$       (b)  $E_y$       (c)  $E_z$       (d)  $H_x$       (d)  $H_y$       (c)  $H_z$

(4) Which of these modes do not exist in a rectangular cavity: (4)

- (a)  $TE_{110}$       (b)  $TE_{011}$       (c)  $TM_{110}$       (d)  $TM_{111}$

(5) In a rectangular waveguide, the cutoff frequency for  $TE_{02}$  is 12 GHz. If  $a = 2b$ , the cutoff frequency for  $TM_{11}$  mode is: (9)

- (a) 3 GHz,      (b)  $3\sqrt{5}$  GHz      (3) 12 GHz,    (c)  $6\sqrt{5}$  GHz    (d) None of these.

Problem 4:

A metallic air filled rectangular waveguide operating at 20 GHz in TE mode has the following magnetic field component:

$$E_x = jE_o \sin\left(\frac{2\pi y}{b}\right) e^{-0.05z - j\beta z}$$

The waveguide delivers 500 Watt power to the antenna located at the end of the waveguide. Given  $a = 5$  cm and  $b = 2$  cm, the length of the waveguide is 100 cm, and  $\sigma_c = 5.8 \times 10^7$  S/m, determine the following

(a) operation mode: (4)

TE<sub>02</sub>

$$f_c = \frac{u}{b} = 15 \text{GHz}$$

(b) The corresponding real electric field for this mode? (6)

$$H_y \neq 0 = \frac{E_x}{\eta} = \frac{jE_o}{\eta} \sin\left(\frac{2\pi y}{b}\right) e^{-0.05z - j\beta z}$$

$$H_y(y, z, w, t) = \frac{E_o}{\eta} e^{-0.05z} \sin\left(\frac{2\pi y}{b}\right) \sin(\omega t - \beta z)$$

(c) The input power  $P_{in}$ : (5)

$$\alpha = 0.05$$

$$P_{in} = P_{out} e^{2\alpha L} = 500 e^{0.1}$$

(d) The power dissipated in the waveguide  $P_d$  (5)

$$P_d = P_{in} - P_{out} = (e^{0.1} - 1) \times 500$$

(e) The power dissipated in the waveguide  $P_d$  when the operation frequency is reduced to 10 GHz? (5)

Frequency  $f=10$  GHz is less than cut-off frequency. No wave propagation. No  $P_d$