

September 28, 2005 Duration: 50 minutes	EE351 Midterm Exam. # 1 (Closed notes & book)	Fall 2005 School of EECS/ WSU
Solution		
Name: _____		ID #: _____

- Answer all questions? Show all intermediate steps for partial credit.
- Closed books
- Closed notes
- Two pages of equations allowed.
- No headphones, cell phones, or pagers.
- No hats
- No guests or visitors during exam.
- See attached equation sheets.

Signature: _____

	Possible Points	Grade point
1	25	
2	25	
3	25	
4	25	
	100	

Problem 1:

The electric field of a uniform plane wave of an electromagnetic field incident vertically on the ocean is given by:

$$\vec{E}(z, t) = \hat{a}_x E_o e^{-\alpha z} \cos(6\pi \times 10^3 t - \beta z) \text{ V/m}$$

where E_o is the electric field amplitude at $z = 0$. The frequency $f = 3 \text{ kHz}$ and the ocean water has the following parameters: $\sigma = 4 \text{ S}\cdot\text{m}^{-1}$, $\epsilon = 81\epsilon_o$, $\mu = \mu_o$. Determine the following: [Hint: first calculate $\frac{\sigma}{\omega\epsilon}$]

(a) Direction of propagation of electromagnetic field: \hat{a}_z (3)

(b) Polarization of the electromagnetic field: \hat{a}_x (3)

(c) Direction of the magnetic field: \hat{a}_y (3)

(d) The propagation constant $\beta =$ (4)

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 3 \times 10^3 \times 81 \times 8.85 \times 10^{-12}} = \frac{4 \times 10^5}{1.35} \gg 1$$

$$\beta = \alpha \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 3 \times 10^3 \times 4\pi \times 10^{-7} \times 4} = 4\pi \times 10^{-2} \sqrt{3} = 0.218 \text{ m}^{-1}$$

(e) The wavelength $\lambda = \frac{2\pi}{\beta} = 28.9 \text{ m}$ (3)

(f) The phase velocity $v = \lambda f = 28.9 \times 3 \times 10^3 = 8.66 \times 10^4 \text{ m/s}$ (3)

(g) The attenuation constant $\alpha = 0.218 \text{ m}^{-1}$ (3)

(h) The skin depth $\delta = 1/\alpha = 4.587 \text{ m}$ (3)

Problem 2:

The phasor expressions for the electric and magnetic fields of an electromagnetic wave in a fluid are given by:

$$\bar{E}(x) = \hat{a}_y 2.84 e^{-0.25x} e^{-j1.571x} \text{ kV/m}$$

$$\bar{H}(x) = \hat{a}_z 36.9 e^{-0.25x} e^{-j1.571x} e^{j\pi/4} \text{ kA/m}$$

- (a) What is the real time form of the electric field? (4)

$$\bar{E}(x) = \hat{a}_y 2.84 e^{-0.25x} \cos(\omega t - 1.57x)$$

- (b) What is the real time form of the magnetic field? (4)

$$\bar{H}(x) = \hat{a}_z 36.9 e^{-0.25x} \cos(\omega t - 1.57x + \pi/4)$$

- (c) What is the time average Poynting vector? (6)

$$P_{ave} = \frac{1}{2} \text{Re}(\bar{E} \times \bar{H}^*) = \hat{a}_x 52.4 \times e^{-0.5x} \cos(\pi/4)$$

- (d) What is intrinsic impedance of the medium η ? (4)

$$\eta = \frac{E}{H} = \frac{2.84}{36.9 \times e^{j\pi/4}} = 0.077 e^{-j\pi/4}$$

- (e) What is the propagation vector γ ? (4)

$$\gamma = \alpha + j\beta = 0.25 + j 1.57$$

- (f) If the wave is incident on a perfect conducting surface at $x = b$, What is the average power at 5 meters to the right of the interface ($x = b + 5$)? (3)

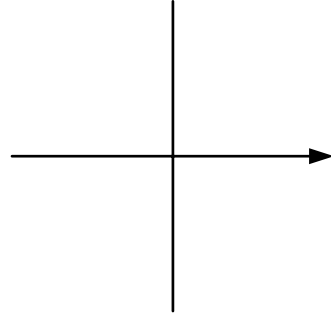
Since there is a perfect conductor to the right of $x = b$, the electric field is zero ($E = 0$)
Therefore $P_{ave} = 0.0$

Problem 3

A plane wave in free space has:

$$\vec{E}_i = 100e^{-j4\pi x} \hat{a}_y \text{ V/m}$$

is normally incident on a lossless medium in region $x > 0$. The reflection coefficient $\Gamma = -0.5$ at the interface $x=0$. Determine the following:



- a) Frequency of incident wave? (3) $\beta = 4\pi \text{ m}^{-1}$; $\lambda = 2\pi/\beta = 1/2 \text{ m}$

Free space, $v = c = 3 \times 10^8 \text{ m/s}$

$$f = c/\lambda = 6 \times 10^8 \text{ Hz}$$

- b) Total electric field \vec{E} in medium 1? (3)

$$\vec{E}_{tot} = \vec{E}_i + \vec{E}_r = 100 (e^{-j4\pi x} + \Gamma e^{j4\pi x}) \hat{a}_y = 100 (e^{-j4\pi x} - 0.5 e^{j4\pi x}) \hat{a}_y$$

- c) Total magnetic field \vec{H} in medium 1? (3)

$$\vec{H}_{tot} = \vec{H}_i + \vec{H}_r = \frac{100}{\eta} (e^{-j4\pi x} - \Gamma e^{j4\pi x}) \hat{a}_z = \frac{100}{377} (e^{-j4\pi x} + 0.5 e^{j4\pi x}) \hat{a}_z$$

- d) Reflection coefficient $\Gamma(-\lambda/4)$ at $x = -\lambda/4$? (4)

$$\Gamma(-\lambda/4) = \Gamma(0) e^{j2\beta z} = -0.5 e^{-j2 \frac{2\pi}{\lambda} \frac{\lambda}{4}} = -0.5 e^{-j\pi} = 0.5$$

- e) Total electric field \vec{E}_2 in medium 2,? (3) $\tau = 1 + \Gamma = 1 - 0.5 = 0.5$

$$\vec{E}_2 = \tau \vec{E}_i = 0.5 \times 100 e^{-j4\pi x} \hat{a}_y = 50 e^{-j4\pi x} \hat{a}_y$$

- f) Intrinsic impedance η_2 in medium 2 (3)

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 0.5, \quad \eta_2 = \frac{\eta_1}{3} = 40\pi$$

- g) The time average power (Poynting vector) in medium (2)? (6)

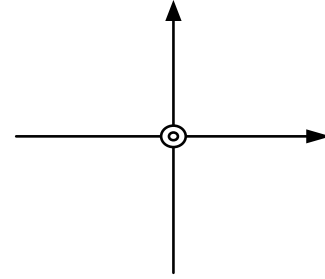
$$P_{ave} = \frac{1}{2} \frac{|E|^2}{\eta_2} \hat{a}_x = \frac{(50)^2}{80\pi} \hat{a}_x \text{ W/m}^2$$

Problem 4:

A plane wave in air has:

$$\vec{H}_i(y, z) = \left(\hat{a}_x 86.2 e^{-j20\pi(0.53y - 0.848z)} \right) \text{ mA/m}$$

is incident on a glass slab in region $y > 0$. Determine the following:



(a) Wave vector β and direction of propagation \hat{a}_k of the incident plane wave? (4)

From equation sheet:

$$\vec{H}_i(y, z) = \left(\hat{a}_x H_{io} e^{-j\beta(y \cos \theta - z \sin \theta)} \right)$$

Comparing with above equation: $\beta = 20\pi$, $\hat{a}_k = 0.53\hat{a}_y - 0.848\hat{a}_z$

(b) Angle of incidence? (3)

$$\cos \theta_i = 0.53, \quad \theta_i = 58^\circ$$

(c) Incident electric field $\vec{E}_i(y, z)$? Sketch on the above plot? (5) Using equation sheet

$$\eta = 377 \Omega$$

$$\vec{E}_i = \eta 86.2 (0.848\hat{a}_y + 0.53\hat{a}_z) e^{-j20\pi(0.53y - 0.848z)} \text{ mV/m}$$

(d) Incident power (time average Poynting vector)? (3)

$$P_{ave} = \frac{1}{2} \eta |H|^2 = \frac{377}{2} (0.0862)^2 = 1.4 \text{ W/m}^2$$

(e) Brewster angle? (3)

Electric field is parallel to the plane of incidence (H is normal to it). Therefore

$$\tan(\theta_B) = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{2.56} = 1.6; \quad \theta_B = 58^\circ$$

(f) Reflected Electric Field $\vec{E}_r(y, z)$? (3)

Angle of incidence is equal to Brewster angle. $\vec{E}_r = 0$. Also reflected power $P_r = 0$

(g) Transmitted power? (3)

From energy conservation: $P_i = P_r + P_t$; $P_t = P_i = 1.4 \text{ W/m}^2$