1 Problem 1

$\bar{A} = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$
$\bar{B} = 2\hat{a}_x + 3\hat{a}_y - 3\hat{a}_z$
Determine the following:

(i) Unit vectors $\hat{a}_A$ and $\hat{a}_B$
$\hat{a} = \frac{\bar{A}}{|\bar{A}|} = \frac{\mathbf{\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}}{\sqrt{\mathbf{14}}}$
$\hat{b} = \frac{\bar{B}}{|\bar{B}|} = \frac{\mathbf{2\hat{a}_x + 3\hat{a}_y - 3\hat{a}_z}}{\sqrt{\mathbf{22}}}$

(ii) $\bar{A} \cdot \bar{B}$
$\bar{A} \cdot \bar{B} = (1 * 2) + (2 * 3) + (3 * -3) = 2 + 6 - 9 = \mathbf{-1}$

(iii) Angle between vectors $\bar{A}$ and $\bar{B}$
$\bar{A} \cdot \bar{B} = |\bar{A}| \, |\bar{B}| \cos\theta$
$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{|\bar{A}||\bar{B}|} = \frac{-1}{\sqrt{14}\sqrt{22}} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{\sqrt{14}\sqrt{22}}\right) = \mathbf{93.27}^\circ$

(iv) Projection of the vector $\bar{A}$ along direction $\hat{b}$
$\bar{A} \cdot \hat{b} = \frac{2+6-9}{\sqrt{22}} = \frac{\mathbf{-1}}{\sqrt{\mathbf{22}}} \; (= -0.2132)$

(v) $\bar{A} \times \bar{B}$
$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & 3 \\ 2 & 3 & -3 \end{vmatrix} = \mathbf{-15\hat{a}_x + 9\hat{a}_y - \hat{a}_z}$$

2 Problem 2

$\bar{A} = b\hat{a}_x + c\hat{a}_y + 3\hat{a}_z$
$\bar{B} = \hat{a}_x - 3\hat{a}_y + \hat{a}_z$

(i) Determine the values of the constants b and c if $\bar{A}$ and $\bar{B}$ are parallel to each other.
$\bar{A} \parallel \bar{B} \Rightarrow \bar{A} \times \bar{B} = 0$
$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ b & c & 3 \\ 1 & -3 & 1 \end{vmatrix} = (c+9)\,\hat{a}_x + (3-b)\,\hat{a}_y - (3b+c)$$
$c + 9 = 0 \Rightarrow \mathbf{c = -9}$
$b - 3 = 0 \Rightarrow \mathbf{b = 3}$
$3b + c = 0 \Rightarrow \mathbf{c = -3b}$ *OKAY!*

(ii) Determine the values of the constants b and c if $\bar{A}$ and $\bar{B}$ are normal to each other.
$\bar{A} \perp \bar{B} \Rightarrow \bar{A} \cdot \bar{B} = 0 = b - 3c + 3 \Rightarrow b - 3c = -3$ or $b = 3c - 3$
Check boundaries: if $c = 0$ then $b = -3$, and if $b = 0$ then $c = 1$.

3 Problem 3

$\bar{E} = xy\hat{a}_x + yz\hat{a}_y - x^2\hat{a}_z$
$\bar{B} = z^2\hat{a}_x - 3xz\hat{a}_y + xy\hat{a}_z$
Determine the following:

(i) $\nabla \cdot \bar{E}$
$\nabla \cdot \bar{E} = \frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z = y + z + 0 = \mathbf{y} + \mathbf{z}$

(ii) $\nabla \cdot \bar{B}$
$\nabla \cdot \bar{B} = 0 - 0 + 0 = \mathbf{0}$ *by Maxwell's eqns*

(iii) $\nabla \times \bar{E}$

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & -xz \end{vmatrix} = -\mathbf{y}\mathbf{\hat{a}_x} + \mathbf{2x}\mathbf{\hat{a}_y} - \mathbf{x}\mathbf{\hat{a}_z}$$

(iv) $\nabla \times \bar{B}$

$$\nabla \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & -3xz & xy \end{vmatrix} = \mathbf{4x}\mathbf{\hat{a}_x} + (\mathbf{2z} - \mathbf{y})\,\mathbf{\hat{a}_y} - \mathbf{3z}\mathbf{\hat{a}_z}$$

4 Problem 4

In free space, the electric field vector is given by $\bar{E}$ =100 $e^{\frac{j\pi}{2}} e^{-\frac{j\pi z}{2}} \hat{a}_y$
Determine the following:

(i) Polarization of the electric field
$\mathbf{\hat{a}_y}$

(ii) Direction of propagation
$+\mathbf{z}$

(iii) Wavelength λ
$\beta = \frac{\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\frac{\pi}{2}} = \mathbf{4m}$

(iv) Frequency
$f = \frac{c}{\lambda} = \frac{3\times 10^8}{4} = \mathbf{75\ MHz}$ (note: $\omega = 2\pi f = 150\pi \times 10^6$ rad/s)

(v) Actual electric field (time domain form)
$E(z,t) = \mathbf{100} \cos\left(\omega \mathbf{t} - \frac{\boldsymbol{\pi} \mathbf{z}}{\mathbf{2}} + \frac{\pi}{\mathbf{2}}\right)\mathbf{\hat{a}_y}$ **V/m**

(vi) Magnetic field vector $\bar{H}$
$\bar{H} = \frac{\bar{E}}{\eta} = -\frac{5}{6\pi} e^{\frac{j\pi}{2}} e^{-j\pi z} \hat{a}_x$ A/m

5 Problem 5

In free space, the electric field vector is given by $\bar{E} = 40\cos(\pi 10^8 t + \beta z)\,\hat{a}_x$
Determine the following:

(i) Polarization of the electromagnetic field
$\hat{\mathbf{a}}_{\mathbf{x}}$

(ii) Direction of propagation
$-\mathbf{z}$

(iii) Wavelength λ
$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} = \frac{2\pi\left(3\times10^8\right)}{\pi 10^8} = \mathbf{6m}$

(vi) Frequency
$f = \frac{\omega}{2\pi} = \frac{\pi 10^8}{2\pi} = \mathbf{50\ MHz}$

(v) Magnetic field vector $\bar{H}$
$\bar{H} = \frac{\bar{E}}{\eta} = -\frac{\mathbf{40}}{\mathbf{120}\pi}\cos(\pi\mathbf{10^8 t} + \beta\mathbf{z})\hat{\mathbf{a}}_{\mathbf{y}}$