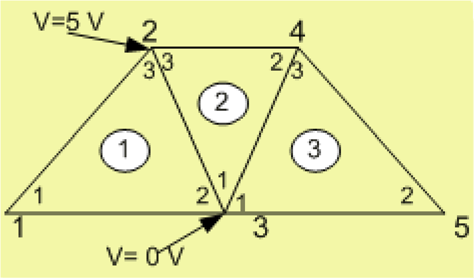
EE351: Homework 10: Problem 1 solution



Problem 1:

If the coordinates of nodes 1 through 5 are (1,1),(3,4),(4,1), (6,4), and (8,1). Determine the global coefficient matrix and solve for the Potentials at the nodes iteratively.

Solution:

(1) Compute element coefficient matrices:

$$\left[C^{(1)}\right]=\begin{bmatrix} 1.6667 & -1.1667 & -0.1667 \\ -1.1667 & 2.1667 & -0.3333 \\ -0.1667 & -0.3333 & 0.1667 \end{bmatrix} \quad \left[C^{(2)}\right]=\begin{bmatrix} 0.3750 & -0.1250 & -0.3750 \\ -0.1250 & 0.4167 & -0.2500 \\ -0.3750 & -0.2500 & 0.7500 \end{bmatrix} \quad \left[C^{(3)}\right]=\begin{bmatrix} 1.0833 & -0.4167 & -0.3333 \\ -0.4167 & 1.0833 & -0.3333 \\ -0.3333 & -0.3333 & 0.3333 \end{bmatrix}$$

(2) Compute the global coefficient matrix:

$$[C]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & C_{42} & C_{43} & C_{44} & C_{45} \\ 0 & 0 & C_{53} & C_{45} & C_{55} \end{bmatrix}$$

$$C_{11}=C_{11}^{(1)};C_{12}=C_{13}^{(1)},C_{13}=C_{12}^{(1)}$$
$$C_{42}=C_{23}^{(2)};C_{43}=C_{21}^{(2)}+C_{31}^{(3)};C_{44}=C_{22}^{(2)}+C_{33}^{(3)},C_{45}=C_{32}^{(3)}$$
$$C_{53}=C_{21}^{(3)};C_{54}=C_{45};C_{55}=C_{22}^{(3)}$$

$$[C]=\begin{bmatrix} 1.6667 & -0.1667 & -1.1667 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.2500 & -0.4583 & 0.7500 & -0.3333 \\ 0.0000 & 0.0000 & -0.4167 & -0.3333 & 1.0833 \end{bmatrix}$$

(3) Iteratively solve the following equations: ε = 0.0001

$$V_1=-\frac{1}{C_{11}}\left[C_{12}V_2+C_{13}V_3\right]$$
$$V_4=-\frac{1}{C_{44}}\left[C_{42}V_2+C_{43}V_3+C_{45}V_5\right]$$
$$V_5=-\frac{1}{C_{55}}\left[C_{53}V_3+C_{54}V_4\right]$$

$$V_1=0.5000V$$
$$V_4=1.9307V$$
$$V_5=0.5941V$$

Global Coefficient Matrix: Effect of Boundary Conditions

$$[C][V]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & C_{42} & C_{43} & C_{44} & C_{45} \\ 0 & 0 & C_{53} & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}=\begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} \\ 0 & 0 & 0 & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}+\begin{bmatrix} C_{12} & C_{13} \\ 1 & 0 \\ 0 & 1 \\ C_{42} & C_{43} \\ 0 & C_{54} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}=0$$

$$\begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} \\ 0 & 0 & 0 & C_{45} & C_{55} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}=-\begin{bmatrix} C_{12} & C_{13} \\ 1 & 0 \\ 0 & 1 \\ C_{42} & C_{43} \\ 0 & C_{54} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

Problem 2 solution

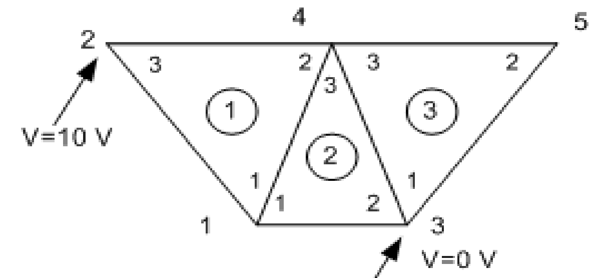


Figure 2.

II. Global matrix is not symmetric: Simplify using:

(1) No direct connection between nodes 1 and 5.	$C_{15} = C_{51} = 0$
(2) No direct connection between nodes 2 and 3	C23 = C32 = 0
(3) No direct connection between nodes 2 and 5	C25 = C52 = 0
(4) voltage at node 2 is fixed at 10V:	C21 = 0 = C23 = C24 = C25 ; C22 = 1
(5) voltage at node 3 is fixed at 0V:	C31 = 0 = C32 = C34 = C35 ; C33 = 1
(6) Nodes 1, 4 and 5 are free nodes:	C14 = C42; C45 = C54

A. Simplified Global Coefficient matrix is:

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\ 0 & 0 & C_{53} & C_{45} & C_{55} \end{bmatrix}$$

$$C_{11} = C_{11}^{(1)}; C_{12} = C_{13}^{(1)}, C_{13} = C_{12}^{(2)}; C_{14} = C_{12}^{(1)} + C_{13}^{(2)}$$

$$C_{42} = C_{23}^{(1)}; C_{43} = C_{32}^{(2)} + C_{31}^{(3)}; C_{44} = C_{22}^{(1)} + C_{33}^{(2)} + C_{33}^{(3)}, C_{45} = C_{32}^{(3)}$$

$$C_{53} = C_{21}^{(3)}; C_{54} = C_{45}; C_{55} = C_{22}^{(3)}$$

B. Solve for V1, V4, and V5 iteratively

$$V_1 = -\frac{1}{C_{11}}\left[C_{12}V_2 + C_{13}V_3 + C_{14}V_4\right]$$

$$V_4 = -\frac{1}{C_{44}}\left[C_{41}V_1 + C_{42}V_2 + C_{43}V_3 + C_{45}V_5\right]$$

$$V_5 = -\frac{1}{C_{55}}\left[C_{53}V_3 + C_{54}V_4\right]$$

Problem 2 solution (continued)

(1) Compute element coefficient matrices:

$$[C^{(1)}] = \begin{bmatrix} -0.5333 & 0.1333 & 0.5333 \\ 0.1333 & -0.3333 & 0.1667 \\ 0.5333 & 0.1667 & -0.8333 \end{bmatrix} \quad [C^{(2)}] = \begin{bmatrix} 1.6667 & -1.3333 & -0.1667 \\ -1.3333 & 1.6667 & -0.1667 \\ -0.1667 & -0.1667 & 0.1667 \end{bmatrix} \quad [C^{(3)}] = \begin{bmatrix} 0.5333 & -0.1333 & -0.5333 \\ -0.1333 & 0.3333 & -0.1667 \\ -0.5333 & -0.1667 & 0.8333 \end{bmatrix}$$

(2) Compute the global coefficient matrix:

$$[C] = \begin{bmatrix} -0.5333 & 0.5333 & -1.3333 & -0.0334 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -0.0334 & 0.1667 & -0.7334 & 0.6667 & -0.1667 \\ 0 & 0 & -0.1333 & -0.1667 & 0.3333 \end{bmatrix}$$

$$C_{11} = C_{11}^{(1)}; C_{12} = C_{13}^{(1)}, C_{13} = C_{12}^{(2)}; C_{14} = C_{12}^{(1)} + C_{13}^{(2)}$$

$$C_{42} = C_{23}^{(1)}; C_{43} = C_{32}^{(2)} + C_{31}^{(3)}; C_{44} = C_{22}^{(1)} + C_{33}^{(2)} + C_{33}^{(3)}, C_{45} = C_{32}^{(3)}$$

$$C_{53} = C_{21}^{(3)}; C_{54} = C_{45}; C_{55} = C_{22}^{(3)}$$

(3) Iteratively solve the following equations: $\varepsilon = 0.0001$

$$V_1 = -\frac{1}{C_{11}}\left[C_{12}V_2 + C_{13}V_3 + C_{14}V_4\right]$$

$$V_4 = -\frac{1}{C_{44}}\left[C_{41}V_1 + C_{42}V_2 + C_{43}V_3 + C_{45}V_5\right]$$

$$V_5 = -\frac{1}{C_{55}}\left[C_{53}V_3 + C_{54}V_4\right]$$

$$V_1 = 5.000V$$

$$V_4 = -1.4286V$$

$$V_5 = 0.7143V$$