1 Problem 1

The complex amplitude of an electric field of a uniform plane wave in free space propagating in the +z direction is given by:
$E_m = 100\hat{a}_x + 20e^{\jmath\frac{\pi}{6}}\hat{a}_y$ V/m
Determine the following assuming a frequency $f = 10$ MHz:

(i) Phasor form of the elctric field
$E = \left[100\hat{a}_x + 20e^{\jmath\frac{\pi}{6}}\hat{a}_y\right]e^{\jmath\beta z}$ V/m

(ii) Time domain (actual) form of the electric field
$E = 100\cos(\omega t - \beta z)\hat{a}_x + 20\cos\left(\omega t - \beta z + \frac{\pi}{6}\right)\hat{a}_y$ V/m

(iii) Phasor form of the magnetic field
$H = \left[\frac{100}{120\pi}\hat{a}_y - \frac{20}{120\pi}e^{\jmath\frac{\pi}{6}}\hat{a}_x\right]e^{\jmath\beta z}$ A/m

(iv) Time domain (actual) form of the magnetic field
$H = \frac{100}{120\pi}\cos(\omega t - \beta z)\hat{a}_y - \frac{20}{120\pi}\cos\left(\omega t - \beta z + \frac{\pi}{6}\right)\hat{a}_x$ A/m

(v) Instantaneous Poynting vector

$$\bar{P}(z,t) = \bar{E}\times\bar{H} = \frac{1}{2}\mathrm{Re}\begin{vmatrix} \hat{a}_y & \hat{a}_y & \hat{a}_z \\ 100\cos(\omega t - \beta z) & 20\cos\left(\omega t - \beta z + \frac{\pi}{6}\right) & 0 \\ -\frac{20}{\eta_0}\cos\left(\omega t - \beta z + \frac{\pi}{6}\right) & \frac{100}{\eta_0}\cos(\omega t - \beta z) & 0 \end{vmatrix}$$

$$\bar{P}(z,t) = \frac{1}{\eta_0}\left[100^2\cos^2(\omega t - \beta z) + 20^2\cos^2\left(\omega t - \beta z + \frac{\pi}{6}\right)\right]\hat{a}_z \text{ W/m}^2$$

(vi) Time average Poynting vector

$$\mathscr{P}_{ave} = \frac{1}{2}\mathrm{Re}\left[\bar{E}\times\bar{H}^*\right] = \frac{1}{2}\mathrm{Re}\begin{vmatrix} \hat{a}_y & \hat{a}_y & \hat{a}_z \\ 100e^{-\jmath\beta z} & 20e^{\jmath\left(\frac{\pi}{6}-\beta z\right)} & 0 \\ -\frac{20}{\eta_0}e^{\jmath\left(-\frac{\pi}{6}+\beta z\right)} & \frac{100}{\eta_0}e^{\jmath\beta z} & 0 \end{vmatrix}$$

$$\mathscr{P}_{ave} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_z\left[\frac{100^2}{\eta_0} + \frac{20^2}{\eta_0}\right]\right] = \frac{10400}{2\eta_0}\hat{a}_z$$

$$\boldsymbol{\mathscr{P}_{ave} = \frac{130}{3\pi}\hat{a}_z \text{ W/m}^2}$$

2 Problem 2

The magnetic field of a plane wave traveling in free space is given by:
$\bar{H} = \left[2e^{-\jmath\frac{2\pi}{9}}\hat{a}_x - 3e^{\jmath\frac{\pi}{9}}\hat{a}_y\right] e^{-\jmath 0.07z}$ A/m
Determine the following:

(i) Propagation direction
+z

(ii) Frequency f
$\beta = \frac{2\pi}{\lambda} = 0.07 \Rightarrow \lambda = \frac{2\pi}{0.07} \Rightarrow f = \frac{c}{\lambda} = (3\times10^8)\left(\frac{0.07}{2\pi}\right) = \mathbf{3.34MHz}$

(iii) H_x at the point $(x, y, z) = (1{,}2{,}3)$ and time $t = 31$ ns.
$H\,(1, 2, 3, 31ns) = 2\cos\left(\omega t - \frac{2\pi}{9} - 0.07z\right)\hat{a}_x = 1.934\hat{a}_x$ A/m

(iv) Magnitude $\left|\bar{H}\right|$ at time $t = 0$ at the origin (0,0,0)
$\left|\bar{H}\right| = \sqrt{\left(2\cos\left(\frac{2\pi}{9}\right)\right)^2 + \left(3\cos\left(\frac{\pi}{9}\right)\right)^2} = 3.21$ A

(v) Time average Poynting vector
$\mathscr{P}_{ave} = \frac{1}{2}\frac{E^2}{|\eta|} = \frac{1}{2}|\eta|\,H^2 = \frac{120\pi}{2}\,(3.21))^2 = \mathbf{1943\ W/m^2}$

3 Problem 3

A 9.375 GHz uniform plane wave polarized along the x-axis is propagating along the z-axis in polyethylene ($\epsilon_r = 2.25, \mu = \mu_0$). If the amplitude of the electric field is 500 V and the material is assumed to be lossless, find the following:

(i) Phasor form of the electric field
$\bar{E} = 500\mathrm{e}^{-\jmath\beta z}\hat{a}_x$ V/m

(ii) Phase velocity
$v_p = 1.99 \times 10^8$ m/s

(iii) Wave vector (propagation constant)
$\gamma = \alpha + \jmath\beta = 0 + \jmath\beta \Rightarrow \gamma = \beta = \omega\sqrt{\mu\epsilon} = 295$ rad/s

(iv) Wavelength λ
$\lambda = \frac{v_p}{f} = 2.13$ cm

(v) Intrinsic impedance
$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{1.5} = 251\Omega$

(vi) Actual electric field (time domain)
$\bar{E} = 100\cos(18.75\pi \times 10^9 t - 295z)\hat{a}_x$ V/m

(vii) Amplitude of the magnetic field vector $\bar{H}$
$\left|\bar{H}\right| = \frac{\left|\bar{E}\right|}{\eta} = (500)\left(\frac{\sqrt{2.25}}{120\pi}\right) = 1.99$ A

4 Problem 4

Starting from the general expressions of α and β in a lossy medium, show that when the loss tangent: $\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon} \gg 1$,

(i) $\alpha = \beta = \sqrt{\pi f \mu \sigma}$

$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} = \omega\sqrt{\frac{\mu\epsilon}{2}\frac{\sigma}{\omega\epsilon}} = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\mu\sigma\pi f}$

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$ =similar reduction to $\alpha = \sqrt{\mu\sigma\pi f}$

(ii) phase velocity $v_p = \omega\delta$

$v_p = \lambda f = \frac{2\pi}{\beta} f = \frac{\omega}{\beta} = \frac{\omega}{\alpha} = \omega\delta$

(iii) Wavelength $\lambda = 2\pi\delta$

$\lambda = \frac{v_p}{f} = \frac{\omega\delta}{f} = \frac{2\pi f\delta}{f} = 2\pi\delta$

5 Problem 5

A plane wave of frequency 1 MHz and power 20 Watts is incident on seawater ($\sigma = 4$ S/m, $\epsilon_r = 81$).
Determine the following:

(i) Loss tangent
$\tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon} = 887.9 \gg 1$

(ii) Skin depth δ
$\delta = \frac{1}{\sqrt{\pi\mu f\sigma}} = 0.25\text{m}$

(iii) Wavelength λ
$\lambda = 2\pi\delta = 1.6\text{m}$

(vi) Phase velocity
$v_p = \omega\delta = (2\pi \times 10^6)(0.25) = 1.6 \times 10^6\text{m/s}$

(v) Power at depth 20 meters
$P(z) = P(0)\,\mathrm{e}^{-\alpha z}; \alpha = \frac{1}{\delta} = \frac{1}{0.25} = 4$
$P(10\text{m}) = P(0)\,\mathrm{e}^{-4*10} =$

What frequency can be used for undersea communication between two submarines that are 80 meters apart?
$\delta = \frac{1}{\alpha} = \frac{1}{\pi f\mu\sigma}$
$f = \frac{1}{\delta^2\pi\mu\sigma} = \frac{1}{80^2\pi*4\pi\times10^7*4} = 9.89\text{Hz}$