EE351 - Spring 2006 - HW3 Solutions

February 8, 2006

Problem 1 - 2 points

In free space, the electric field vector given by $\bar{E}(z,t) = 10\cos(2\pi \times 10^8 t - \beta z)\hat{a}_x$ V/m is incident on a 20 cm diameter receiving antenna dish. Determine the following:

(i) Amplitude of the electric field
$\left|\bar{\mathbf{E}}\right| = \mathbf{10}\text{V}$

(ii) Time average Poynting vector
$\mathscr{P}_{ave} = \frac{1}{2}\frac{10^2}{120\pi}\hat{a}_z = 0.133\hat{a}_z$ mW/m^2

(iii) The power incident on the dish
$P = \oint \mathscr{P}_{ave} dS = \left(1.3\frac{mW}{m^2}\right)\left(\frac{\pi(0.2m)^2}{4}\right) = 4.17\text{mW}$

(iv) Magnetic field vector
$\bar{H} = \frac{10}{\eta}\cos(2\pi \times 10^8 t - \beta z)\hat{a}_y$ A/m

Problem 2 - 2 points

A thick slab of polystyrene ($\sigma = 10^6 S/m; \epsilon_r = 2.6$) occupies $z > 0$. If at the surface of the slab ($z = 0$), the electric field $\bar{E}\,(0, t) = 10 \cos{(3\pi \times 10^7 t)} \hat{a}_y$ Determine the following:

(i) Electric field vector $\bar{E}\,(z, t)$
$\bar{E}\,(z, t) = 10\mathrm{e}^{-\alpha z} \cos{(3\pi \times 10^7 t - \beta z)} \hat{a}_y$ V/m

(ii) Magnetic field vector $\bar{H}\,(z, t)$
$\bar{H}\,(z, t) = -\frac{10}{\eta}\mathrm{e}^{-\alpha z} \cos{(3\pi \times 10^7 t - \beta z)} \hat{a}_x$ A/m

(iii) Time average Poynting vector
$\mathscr{P}_{\text{ave}} = \frac{1}{2}\frac{|E|^2}{\eta} = 210\mathrm{e}^{-2\alpha z} \hat{a}_y$ mW/m^2

(iv) Frequency f
$f = \frac{\omega}{2\pi} = 15$MHz

(v) Wave vector
$\beta = \omega\sqrt{\epsilon\mu} = 0.507$rad/m; $\alpha = 12 \times 10^{-15}$Np/m

Problem 3 - 2 points

The plane z = 0 separates two lossless, non-magnetic media. Medium 1 ($z < 0$) has $\epsilon_r = 4$ and medium 2 ($z > 0$) is air. If the incident electric field is given by:
$\bar{E}_i(z,t) = 10\cos(\omega t - \beta_1 z)\hat{a}_x$
Determine the following:

(i) Intrinsic impedances η_1 and η_2
$\eta_1 = 60\pi\Omega, \eta_2 = 120\pi\Omega$

(ii) The incident fields $\bar{E}_i(z,t)$ and $\bar{H}_i(z,t)$
$\bar{E}_i = 10\cos(\omega t - \beta_1 z)\hat{a}_x$ V/m
$\bar{H}_i = \frac{10}{\eta_1}\cos(\omega t - \beta_1 z)\hat{a}_y$ A/m

(iii) Reflection and transmission coefficients
$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{120\pi - 60\pi}{120\pi + 60\pi} = \frac{1}{3}$
$\tau = 1 + \Gamma = \frac{4}{3}$

(iv) The reflected fields $\bar{E}_r(z,t)$ and $\bar{H}_r(z,t)$
$\bar{E}_r = \frac{10}{3}\cos(\omega t - \beta_1 z)\hat{a}_x$ V/m
$\bar{H}_r = \frac{10}{3\eta_1}\cos(\omega t - \beta_1 z)\hat{a}_y$ A/m

(v) The transmitted fields $\bar{E}_t(z)$ and $\bar{H}_t(z)$
$\bar{E}_t = \frac{40}{3}\cos(\omega t - \beta_2 z)\hat{a}_x$ V/m
$\bar{H}_t = \frac{40}{3\eta_2}\cos(\omega t - \beta_2 z)\hat{a}_y$ A/m

(vi) Incident time average power density
$P^i_{ave} = \frac{1}{2}\text{Re}[E_i \times H_i] == 265\hat{a}_z$ mW/m^2

(vii) Reflected time average power density
$P^r_{ave} = \frac{1}{2}\text{Re}[E_r \times H_r] = -29.5\hat{a}_z$ mW/m^2

(viii) Transmitted time average power density
$P^t_{ave} = \frac{1}{2}\text{Re}[E_t \times H_t] = 235\hat{a}_z$ mW/m^2

Problem 4 - 1 point

Calculate the skin depth at 1 GHz for the following elements:
$\delta = \frac{1}{\pi \mu f \sigma}$

(a) copper
$\delta = 2.09\mu$m

(b) silver
$\delta = 2.02\mu$m

(c) gold
$\delta = 2.49\mu$m

(d) nickel
$\delta = 168$nm

Material	Conductivity σ (S/m)	μ_r	ϵ_r
Copper	5.8×10^7	1.0	1.0
Gold	4.1×10^7	1.0	1.0
Silver	6.2×10^7	1.0	1.0
Nickel	1.5×10^7	600	1.0

Problem 5 - 2 points

The electric field $\bar{E}_i\left(z,t\right) = \cos\left(2\pi \times 10^8 t - \beta_1 z\right)\hat{a}_x$ V/m is incident from air ($z < 0$) onto a non-mangetic lossy medium ($z > 0$) characterized by $\sigma = 10^-2$ S/m; $\epsilon_r = 2.0$. Determine the following:

(i) Wave vector β_1 in air
$\beta_1 = \frac{2\pi}{3} = 2.1$ rad/s

(ii) Loss tangent in medium 2
$\tan\left(2\theta_\eta\right) = \frac{\sigma}{\omega\epsilon} = 0.899$

(iii) Intrinsic impedances η_1 and η_2
$\eta_1 = 120\pi\Omega$
$\eta_2 = 230\mathrm{e}^{\jmath 21^\circ}\Omega$

(vi) Reflection and transmission coefficients
$\Gamma = \frac{\eta_1-\eta_2}{\eta_1+\eta_2} = \frac{120\pi-230\angle 21^circ}{120\pi+230\angle 21^\circ} = -0.30\mathrm{e}^{\jmath 145^\circ}$
$\tau = 1 + \Gamma = 0.77\mathrm{e}^{\jmath 13^\circ}$

(v) The reflected fields $\bar{E}_r\left(z\right)$ and $\bar{H}_r\left(z\right)$
$\bar{E}_r = \Gamma\bar{E}_i = 3\cos\left(2\pi \times 10^8 t - \frac{2\pi}{3}z + 145^\circ\right)\hat{a}_x$ V/m
$\bar{H}_r = \Gamma\frac{\bar{E}_i}{\eta_1} = \frac{3}{120\pi}\cos\left(2\pi \times 10^8 t - \frac{2\pi}{3}z + 145^\circ\right)\hat{a}_y$ A/m

(vi) The transmitted fields $\bar{E}_t\left(z\right)$ and $\bar{H}_t\left(z\right)$
$\bar{E}_t = \tau\bar{E}_i = 7.7\mathrm{e}^{-1.2z}\cos\left(2\pi \times 10^8 t - 3.2z + 13^\circ\right)\hat{a}_x$ V/m
$\bar{H}_t = \tau\frac{\bar{E}_i}{\eta_2} = \frac{7.7}{230\angle 21^\circ}\mathrm{e}^{-1.2z}\cos\left(2\pi \times 10^8 t - 3.2z + 8^\circ\right)\hat{a}_y$ A/m

(vii) Incident time average power density
$P^i_{ave} = \frac{1}{2}\frac{|E_i|^2}{\eta_1} = 132\hat{a}_z$ mW/m^2

(viii) Transmitted time average power density
$P^t_{ave} = 120\mathrm{e}^{-2.4z}\hat{a}_z$ mW/m^2

Problem 6 - 1 point

(a) What is the polarization and tilt angle of:
$\bar{E}_i\left(z,t\right) = 10\cos\left(\omega t - \beta_1 z\right)\hat{a}_x + 5\cos\left(\omega t - \beta_1 z + 270°\right)\hat{a}_y$

Linear, $\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.6°$

(b) What is the polarization of the following fields:

(i) $\bar{E}_i\left(z,t\right) = 10\cos\left(\omega t - \beta_1 z\right)\hat{a}_x + 10\cos\left(\omega t - \beta_1 z + 270°\right)\hat{a}_y$
Circular

(ii) $\bar{E}_i\left(z,t\right) = 10\cos\left(\omega t - \beta_1 z\right)\hat{a}_x + 20\cos\left(\omega t - \beta_1 z + 270°\right)\hat{a}_y$
Ellipse