Problem 1

In free space, a 3 GHz plane wave with the electric field vector given by:
\[ \vec{E}_i(y, z) = E_{m0}e^{-j\beta_1(y\cos\theta_i-z\sin\theta_i)}\hat{a}_x \text{ V/m} \]
is obliquely incident on the \( y = 0 \) interface between air and polystyrene
\((\sigma_2 = 0, \varepsilon_2 = 2.56, \mu_2r = 1)\) as shown below. If the time average power is
1.4 W/m² and the angle of incidence is 58°, determine the following:

(i) Amplitude of the electric field \( E_{m0} \), propagation vector magnitude \( \beta_1 \)

\[ |P_{ave}| = \frac{E_{m0}^2}{2\eta_1} = 1.4 \text{ W/m}^2; \quad \eta_1 = 377 \]

\[ E_{m0} = \sqrt{\frac{2\eta_1 \times 1.4}{\eta_1}} = 32.5 \text{ V} \]

\[ \beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = \frac{\omega}{c} = \frac{2\pi \times 3 \times 10^9}{3 \times 10^8} = 20\pi \text{ rad/m} \]

\[ \beta_i = \beta_1 \cos\theta_i\hat{a}_x - \beta_1 \sin\theta_i\hat{a}_z; \quad \hat{a}_{ki} = \frac{\beta_i}{\beta_1} = \cos\theta_i\hat{a}_x - \sin\theta_i\hat{a}_z \]

(ii) Incident magnetic field \( \vec{H}_i(y, z) \)

\[ \vec{H}_i = \frac{1}{\eta_1}\hat{a}_k \times \vec{E}_i(y, z) = \frac{1}{\eta_1} (\cos\theta_i\hat{a}_x - \sin\theta_i\hat{a}_z) \times \hat{a}_x E_{m0}e^{-j\beta_1(y\cos\theta_i-z\sin\theta_i)} \]

\[ \vec{H}_i = -86.2 [0.848\hat{a}_y + 0.53\hat{a}_z] e^{-j20\pi(0.53y-0.848z)} \text{ mA/m} \]

(iii) Reflected electric field

To find \( \Gamma_\perp \), we need \( \theta_t \) → Use Snell’s Law \((n_1 \sin\theta_i = n_2 \sin\theta_t)\)

\[ \sin58^\circ = \sqrt{2.56}\sin\theta_t \Rightarrow \theta_t = 32^\circ \]

\[ \Gamma_\perp = \frac{\cos58^\circ - \sqrt{2.56}\cos32^\circ}{\cos58^\circ + \sqrt{2.56}\cos32^\circ} = -0.438 \]

\[ E_r = \Gamma_\perp E_{m0}e^{j\beta_1(1 \cos\theta_r + 2 \sin\theta_r)}\hat{a}_x \]

\[ \vec{E}_r = -14.2 e^{20\pi(0.53y+0.848z)}\hat{a}_x \text{ V/m} \]
(iv) Reflected magnetic field
\[ \hat{a}_E \times \hat{a}_H = \hat{a}_{kr} \Rightarrow \hat{a}_H = \hat{a}_{kr} \times \hat{a}_E \]
\[ \hat{a}_{kr} \times \hat{a}_E = [-0.53\hat{a}_y - 0.848\hat{a}_z] \times [-\hat{a}_x] = [0.53\hat{a}_y - 0.848\hat{a}_z] \]
\[ \bar{H}_r = -37.8 [0.848\hat{a}_y - 0.53\hat{a}_z] e^{20\pi(0.53y-0.848z)} \text{ mA/m} \]

(v) Time average reflected power
\[ P_{ave}^r = \frac{1}{2} \Re [E_r \times H_r^*] = \frac{E_r^2}{2\eta_1} = \frac{14.2^2}{2(377)} = 0.269 \text{ W/m}^2 \]

(vi) Time average transmitted power
\[ \tau_\perp = \frac{\cos 58^\circ + \sqrt{2.56\cos 32^\circ}}{2\cos 58^\circ} = 0.562 \leftrightarrow (1 + \Gamma_\perp = \tau_\perp) \]
\[ P_{ave}^t = \frac{(0.562+32.5)^2}{2\times236} = 0.707 \text{ W/m}^2 \]
Problem 2

A 10 W/m², 200 Mhz uniform plane wave propagating in a lossless medium has an electric field vector given by:

\[ E_i (y, z) = \frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x+y)} \hat{a}_x - \frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x+y)} \hat{a}_y \text{ V/m} \]

is obliquely incident upon a perfectly conducting surface located in the xz plane as shown below. Determine the following:

(i) The propagation vector \( \vec{\beta}_1 \)

\[ \vec{\beta}_1 \cdot \vec{r} = x\sqrt{\pi} + y\sqrt{\pi} = \beta_{1x}x + \beta_{1y}y + \beta_{1z}z \]

\[ \beta_{1x} = \sqrt{\pi}; \beta_{1y} = \sqrt{\pi}; \beta_{1z} = 0 \]

\[ \vec{\beta}_1 = \sqrt{\pi} \hat{a}_x + \sqrt{\pi} \hat{a}_y; \beta_1 = \sqrt{2\pi} \text{ rad/m} \]

(ii) The direction of propagation \( \vec{a}_{ik} \)

\[ \vec{a}_{ik} = \frac{\vec{\beta}_1}{\beta_1} = \frac{1}{\sqrt{2}} \hat{a}_x + \frac{1}{\sqrt{2}} \hat{a}_y \]

(iii) The angle of incidence \( \theta_i \)

\[ \hat{a}_n = \hat{a}_y; \cos \theta_i = \hat{a}_{ki} \cdot \hat{a}_n = \frac{1}{\sqrt{2}} \hat{a}_x + \frac{1}{\sqrt{2}} \hat{a}_y \cdot \hat{a}_y = \frac{1}{\sqrt{2}} \]

\[ \theta_i = 45^\circ \]

(iv) The dielectric constant of the lossless medium

\[ \beta_1 = \omega \sqrt{\mu_1 \varepsilon_1} = \omega \sqrt{\mu_0 \varepsilon_0 \sqrt{\varepsilon_r}} \]

\[ \varepsilon_r = \frac{\frac{9 \times 2\pi}{4 \times (2\pi)^2}}{\frac{9}{8\pi}} = 0.36 \text{ (Note: Not realistic number)} \]

(v) Amplitude of the electric field \( E_m \)

\[ P_{ave} = \frac{|E_1|^2}{2\eta_1} = \frac{|E_m|^2}{2\eta_1} + \frac{|E_m|^2}{2\eta_1} = \frac{|E_m|^2}{2\eta_1} = 10 \text{ W/m}^2 \]

\[ E_m = \sqrt{2\eta_1 \times 10}; \eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{0.6} = 628.3 \]

\[ E_m = 112.1 \text{ V} \]

(vi) The reflected electric field

\[ E_y = E_{||}; E_{x1} + E_{xr} = 0 \]

\[ E_{x1} = \frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x+y)} \hat{a}_x; E_{xr} = -\frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x-y)} \hat{a}_x \]

\[ E_r = -\frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x-y)} \hat{a}_x - \frac{E_m}{\sqrt{2}} e^{-j\sqrt{\pi}(x-y)} \hat{a}_y \text{ V/m} \]

(vii) Polarization of the incident and reflected waves

Both the incident and reflected waves are **linearly polarized**. There is 0 phase difference between the \( \hat{a}_x \) and \( \hat{a}_y \) components.
Problem 3

Determine the refractive index and minimum thickness of a film to be deposited on the glass surface \( n_3 = 1.52 \) such that no normally incident visible light from free space (\( \lambda = 550\text{nm} \)) is reflected. What will be the reflection coefficient if magnesium flouride \( n = 1.38 \) was used instead of the film.

Refractive index \( n_2 \) & minimum thickness \( d \)

\[
\eta_2 = \sqrt{\eta_1 \eta_3} \\
n_1 = 1; n_3 = \sqrt{\epsilon_r} \Rightarrow \epsilon_r = n_3^2 = 2.31 \\
\eta_3 = \nu \frac{\eta_1}{\sqrt{\epsilon_r}} = \frac{\eta_0}{n_3} = 248\Omega = \eta_3 \\
n_2 = \sqrt{337 \times 248} = 305.8\Omega = \frac{n_0}{n_2} \\
n_2 = \frac{n_0}{\eta_2} = \frac{377.305.8}{1.23} = 301\Omega \\
d = \frac{\lambda_3}{4} \text{; visible light has } \lambda = 550 \text{ nm in free space} \\
\lambda_3 = \frac{\lambda_2}{n_2} = \frac{550 \times 10^{-9}}{1.23} = 0.448\mu\text{m} \\
d = \frac{0.448}{4} = 0.112\mu\text{m}
\]

Using magnesium flouride \( n = 1.38 \neq 1.23 \), assume \( d = \frac{\lambda_2}{4} \), follow the procedure discussed in class (Find \( Z_2 (-d) \)).

\[
Z_2 (-d) = \eta_2 \frac{\eta_0 + \eta_2 \tan (\beta_2 d)}{\eta_2 + \eta_0 \tan (\beta_2 d)} \\
d = \frac{\lambda_2}{4}; \beta_2 = \frac{2\pi}{4}; \tan (\beta_2 d) = \tan \frac{\pi}{4} = \infty \\
\Rightarrow Z_2 (-d) = \frac{\eta_2}{\eta_3} \text{; But } \eta_3 = \frac{\eta_0}{n_3} = 248\Omega; \eta_2 = \frac{377}{1.38} = 273 \\
Z_2 (-d) = 301\Omega = \eta_{eH} \text{ [for media 2 & 3]} \\
\Gamma_{eH} = \frac{\eta_{eH} - n_1}{\eta_{eH} + n_1} = \frac{301 - 377}{301 + 377} = -0.112 = \Gamma_{eH}
Problem 4 - 8.15

The magnetic field intensity in a medium is given by:
\[ \vec{H} = 0.1e^{-77.485y} \cos (2\pi \times 10^9 t - 203.8y) \hat{a}_x \text{ A/m} \]

If the medium is characterized by the free space permeability, determine the dielectric constant and the conductivity of the medium. Obtain the associated component of the \( \vec{E} \) field. Compute the average power density.

Find \( \sigma \) & \( \epsilon_r \):
Comparing with the general equation for \( \vec{H} \), get can extract the following terms:
\[ \alpha = 77.485; \beta = 203.8; \omega = 2\pi \times 10^9 \text{ rad/s} \]
We know, \( \gamma^2 = (\alpha + j\beta)^2 = j\omega\mu (\sigma + j\omega\epsilon) \Rightarrow \alpha^2 + \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon \)
This yields two relationships, \( 2\alpha\beta = \omega\mu\sigma \) and \( \omega^2\mu\epsilon = \beta^2 - \alpha^2 \)
Solve for \( \sigma \): \( \sigma = \frac{2\alpha\beta}{\omega\mu} = \frac{2 \times 77.485 \times 203.8}{2\pi \times 10^6 \times 4\pi \times 10^{-7}} = 4 \text{ S/m} \)
\( \epsilon_r = \frac{\beta^2 - \alpha^2}{\omega^2\mu\epsilon} = \frac{(203.8)^2 - (77.485)^2}{(2\pi \times 10^9)^2} = 80.8 = \epsilon_r \)

Find \( \vec{E} \) & \( P_{ave} \)
\( \vec{E} = \eta \vec{H} \); \( \eta \) is complex
\[ \eta = \sqrt{\frac{\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| e^{j\theta_{\eta}} \]
\[ |\eta| = \sqrt{\frac{\frac{\omega\mu}{\sigma + j\omega\epsilon}}{1 + (\frac{\epsilon_r}{\epsilon})}} \]
\[ \tan 2\theta_{\eta} = \frac{4}{2\pi \times 10^9 \times 80.8 \times 8.85 \times 10^{-12}} = \frac{8}{9} = 0.88 \Rightarrow \theta_{\eta} = \frac{1}{2} \tan^{-1} 0.88 = 20.8^\circ \]
\[ |\eta| = \sqrt{\frac{\frac{\omega\mu}{\sigma + j\omega\epsilon}}{1 + (\frac{\epsilon_r}{\epsilon})}} = \frac{377}{1.046 \sqrt{80.8}} = 36.2 \Omega \]
\( \hat{a}_k \times \hat{a}_H = \hat{a}_x; \hat{a}_E \times \hat{a}_x = \hat{a}_y \Rightarrow \hat{A}_E = \hat{a}_x \)
\( \vec{E} = \eta \vec{H} \hat{a}_x = 3.62e^{-77485y} \cos (2\pi \times 10^9 t - 203.8y + \theta_{\eta}) \hat{a}_x \text{ V/m} \)
\( P_{ave} = \frac{1}{2} [E \times H^*] = \frac{1}{2} \times 3.62 \times 0.1e^{-2+77.485y} \cos 20.8^\circ \)
\( P_{ave} = 0.169e^{-154.97y} \hat{a}_y \text{ W/m}^2 \)
Problem 5 - 8.17

A uniform plane wave is propagating in a good conductor. If the magnetic field intensity is given by:
\[ \hat{H} = 0.1e^{-15z} \cos (2\pi \times 10^8 t - 15z)\hat{a}_x \text{ A/m} \]
determine the conductivity and the corresponding component of the \( \vec{E} \) field.
Calculate the average power loss in a block of unit area and \( \delta \) thickness.

Find \( \sigma \) & \( \delta \)
Comparing with the general equation for \( \hat{H} \), get can extract the following terms:
\[ \alpha = 15; \beta = 15; \omega = 2\pi \times 10^8 \text{ rad/s}; \alpha = \beta = \sqrt{\pi \mu f \sigma} \]
\[ \sigma = \frac{\alpha^2}{\pi \mu f} = \frac{225}{\pi \times 4 \pi \times 10^{-7} \times 10^8} = 0.57 \text{ S/m} = \sigma \]
\[ \delta = \frac{1}{\alpha} = \frac{1}{15} = 0.067 \text{m} = \delta \]

Find \( \vec{E} \) & Power loss for a unit area
Assume \( \epsilon = \epsilon_0 \)
\[ \omega \epsilon = \frac{0.57}{2\pi \times 10^8 \times 8.85 \times 10^{-12}} = \frac{57}{2\pi \times 8.85} = 1.025 \]
\[ \theta_\eta = \frac{1}{2} \tan^{-1}(1.025) = 22.85^\circ \]
\[ |\eta| = \frac{377}{1.26} = 315\Omega \text{ (For more detail, see the solution for Problem 4 - 8.15)} \]
\[ \vec{E}(z, t) = -31.5e^{-15z} \cos (2\pi \times 10^8 t - 15z + \theta_\eta)\hat{a}_y \text{ V/m} \]
\[ P_{\text{ave}} = \frac{1}{2} [E \times H^*] = \frac{1}{2} \times 0.1 \times 31.5e^{-30z} \cos 22.85^\circ = 1.45e^{-30z} \text{ W/m}^2 \]
At \( z = 0 \), \( P_{\text{ave}} (0) = 1.45 \); At \( z = \delta \), \( P_{\text{ave}} (\delta) = 1.45e^{-2} = 1.45 \times 0.135 \)
Power Loss = \( P_{\text{ave}} (0) - P_{\text{ave}} (\delta) = 1.45 [1 - 0.135] \times 1\text{m}^2 \)
Power Loss = \( 1.254 \text{W} \)
Problem 6 - 8.22

Find the polarization of the following waves:

a) \[ \vec{E} = 100e^{-j300x}\hat{a}_y + 100e^{-j300x}\hat{a}_z \text{ V/m} \]
   \[ \vec{E} = \vec{E}_y\hat{a}_y + \vec{E}_z\hat{a}_z \] (Notice \( E_y \) and \( E_z \) are in phase)
   \[ \frac{E_y}{E_z} = \frac{100\cos(\omega t - 300x)}{100\cos(\omega t - 300x)} = 1 \Rightarrow \text{Linear Polarization} \]
   \[ E(x,t) = [100\cos(\omega t - 300x)](\hat{a}_y + \hat{a}_z) \]

b) \[ \vec{E} = 16e^{j\frac{\pi}{4}}e^{-j100z}\hat{a}_x - 9e^{-j\frac{\pi}{4}}e^{-j100z}\hat{a}_y \text{ V/m} \]
   \[ E(z,t) = 16\cos(\omega t - 100z + \frac{\pi}{4})\hat{a}_x - 9\cos(\omega t - 100z - \frac{\pi}{4})\hat{a}_y \]
   \[ E_x = 16\cos(\omega t - 100z + \frac{\pi}{4})\hat{a}_x \]
   \[ E_y = -9\cos(\omega t - 100z - \frac{\pi}{4})\hat{a}_y \]
   - Amplitudes of \( E_x \) and \( E_y \) are difference \((E_{mx} \neq E_{my})\)
   - Phase difference = \( \frac{\pi}{2} \) \Rightarrow Elliptical Polarization

c) \[ \vec{E} = 3\cos(t - 0.5y)\hat{a}_x - 4\sin(t - 0.5y)\hat{a}_z \text{ V/m} \]
   \[ E_x = 3\cos(t - 0.5y)\hat{a}_x \]
   \[ E_z = -4\sin(t - 0.5y)\hat{a}_z \]
   - Amplitudes of \( E_x \) and \( E_z \) are difference \((E_{mx} \neq E_{mz})\)
   - Phase difference = \( \frac{\pi}{2} \) \Rightarrow Elliptical Polarization