Problem 1:
An air filled waveguide with dimensions a = 5 cm and b= 2 cm, at f=15 GHz has

$$E_z = 20 \sin 40\pi x \sin 50\pi y e^{-j\beta z} V/m$$

(i) the mode of propagation

Using a =0.05 m and b= 0.02 m, and comparing to:

$$E_z = 20\sin\frac{m\pi}{0.05}x\sin\frac{n\pi}{0.02}ye^{-j\beta z}$$

$$E_z = 20\sin(20m\pi x)\sin(50n\pi y)e^{-j\beta z}$$

we find m = 2 and n=1. Mode of propagation is TM_{21}

(ii) Value of β

$$\beta = \beta_{21} = \frac{\omega}{v_p}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi x 15x10^9}{3x10^8}\sqrt{1-\left(\frac{9.6}{15}\right)^2} = 241.3 rad/m$$

(iii) Cut off frequency

$$f_c = \frac{c}{2}\sqrt{\left(\frac{2}{0.05}\right)^2 + \left(\frac{1}{0.02}\right)^2} = \frac{15GHz}{5}\sqrt{4+\left(\frac{5}{2}\right)^2} = 9.6GHz$$

(iv) Intrinsic wave impedance η

$$\eta = \eta_{21} = \eta_o\sqrt{1-\left(\frac{9.6}{15}\right)^2} = 222.5\Omega$$

(v) E_x (see handout).

Problem 2: (problem 10.9)

$$\lambda_{21} = \frac{2\pi}{\beta_{21}} = \frac{2\pi}{165} = 3.81cm$$

To determine the wavelength in unbounded medium λ use the following relationship.

$$\beta_{mn} = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\lambda = \lambda_{mn}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3.81\sqrt{1-\left(\frac{f_c}{1.1f_c}\right)^2} = 1.59cm$$

Problem 3: (Ex. 10.1):

Air filled WG: l=2m, a=2cm, b=1cm, TM_{11}, β=200 m^{-1}

$$f_c = \frac{c}{2}\sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.01}\right)^2} = 6.71GHz$$

$$\beta_{11} = \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$200 = \frac{2\pi f}{3x10^8}\sqrt{1-\left(\frac{6.71x10^9}{f}\right)^2}$$

This gives f = $1.16x10^{10}$ Hz.

Exercise 10.2

At $z = 0$ $\quad E_{zm} = 2\,kV/m$, $\quad \hat{\gamma}_{11} = j200$, $\quad f = 1.167\times10^{10}\,Hz$, $\mu=\mu_0$, $\epsilon=\epsilon_0$

$$M = \frac{\pi}{2\times10^{-2}} = 50\pi \quad , \quad N = \frac{\pi}{10^{-2}} = 100\pi$$

From Eqs. (10.16) and (10.17),

$$\tilde{E}_z = 2000 \sin(50\pi x)\sin(100\pi y)\, e^{-j200z}$$

$$E_z(t) = 2000 \sin(50\pi x)\sin(100\pi y)\cos(2\pi\times1.167\times10^{10}t - 200z)$$

$$\tilde{E}_x = \frac{-j200}{(50\pi)^2+(100\pi)^2}\, 50\pi \times 2000 \cos(50\pi x)\sin(100\pi y)\, e^{-j200z}$$

$$\tilde{E}_x = -j5.093\times10^2 \cos(50\pi x)\sin(100\pi y)\, e^{-j200z}$$

$$E_x(t) = 509.3 \cos(50\pi x)\sin(100\pi y)\cos\left(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2}\right)$$

$$\tilde{E}_y = -\frac{j200}{(50\pi)^2+(100\pi)^2}(100\pi)(2000)\sin(50\pi x)\cos(100\pi y)\, e^{-j200z}$$

$$\tilde{E}_y = -j1018.59 \sin(50\pi x)\cos(100\pi y)\, e^{-j200z}$$

$$E_y(t) = 1018.59 \sin(50\pi x)\cos(100\pi y)\cos\left(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2}\right)$$

$$\tilde{H}_x = \frac{j1.167\times10^{10}\times2\pi\times8.85\times10^{-12}}{(50\pi)^2+(100\pi)^2}\, 100\pi\times2000 \sin(50\pi x)\cos(100\pi y)\, e^{-j200z}$$

$$\tilde{H}_x = j3.30 \sin(50\pi x)\cos(100\pi y)\, e^{-j200z}$$

$$H_x(t) = 3.30 \sin(50\pi x)\cos(100\pi y)\cos\left(2\pi\times1.167\times10^{10}t - 200z + \frac{\pi}{2}\right)$$

$$\tilde{H}_y = -\frac{j2\pi\times1.167\times10^{10}\times8.85\times10^{-12}}{(50\pi)^2+(100\pi)^2}\, 50\pi\times2000 \cos(50\pi x)\sin(100\pi y)\, e^{-j200z}$$

$$\tilde{H}_y = -j1.65 \cos(50\pi x)\sin(100\pi y)\, e^{-j200z}$$

$$H_y(t) = 1.65 \cos(50\pi x)\sin(100\pi y)\cos\left(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2}\right)$$

Problem 5: (Ex. 10.3):

For a= 1cm, b=0.5 cm, $\varepsilon = 2.5\,\varepsilon_0$, Ezm = 1.5 kV/m at z=0, TM_{11} and f=9 GHz

$$v_p = \frac{1}{\sqrt{2.5\mu_o\varepsilon_o}} = 1.897x10^8\, m/s$$

$$f_{c11} = \frac{1.897x10^{10}}{2}\sqrt{1+\left(\frac{1}{0.5}\right)^2} = 21.21GHz \rangle 9GHz$$

No propagation. Only evanescent modes exist. (questions b through e are irrelevant)

Using a = 2.5 cm and b=1.25 cm, the cut of frequency is:

(a) $$f_{c11} = \frac{1.897x10^{10}}{2}\sqrt{\left(\frac{1}{2.5}\right)^2 + \left(\frac{1}{1.25}\right)^2} = 8.48GHz < 9GHz$$

b) $$\beta_{11} = \frac{2\pi \times 9\times10^9}{1.897\times10^8}\sqrt{1-\left(\frac{8.48\times10^9}{9\times10^9}\right)^2} = 99.52 \text{ rad/m}$$

$$\hat{\gamma}_{11} = j\,99.52$$

c) $$u_{p_{11}} = \frac{\omega}{\beta_{11}} = \frac{2\pi \times 9\times10^9}{99.52} = 5.68\times10^8 \text{ m/s}$$

$$u_{g_{11}} = \frac{(1.897\times10^8)^2}{5.68\times10^8} = 6.333\times10^7 \text{ m/s}$$

d) $$\eta = \sqrt{\frac{4\pi\times10^{-7}}{2.5\times8.85\times10^{-12}}} = 238.32\ \Omega$$

$$\eta_{11} = 238.32\sqrt{1-\left(\frac{8.48\times10^9}{9\times10^9}\right)^2} = 79.83\ \Omega$$

e) From Eq. (10.50),

$$\langle P_{11}\rangle = \frac{99.52^2\times(10^{-2})^3(5\times10^{-3})^3}{8\pi^2\times79.83\left[(10^{-2})^2+(5\times10^{-3})^2\right]}\times1500^2$$

$$\langle P_{11}\rangle = 3.53\times10^{-3}\text{ W} \quad \text{or} \quad 3.53\text{ mW}$$