Exercise 10.4

$b = 1\,cm, \quad f = 12\,GHz, \quad \beta_{10} = 150\,rad/m \quad TE_{10}.$

$$150 = \frac{2\pi \times 12 \times 10^9}{3 \times 10^8}\sqrt{1-\left(\frac{f_{c10}}{12}\right)^2} \Rightarrow f_{c10} = 9.63\,GHz$$

$$f_{c10} = \frac{u_p}{2a} \Rightarrow a = \frac{3\times10^8}{2\times9.63\times10^9} = 1.557\times10^{-2}\,m \text{ or } a = 1.557\,cm$$

Exercise 10.5

$a = 1\,cm, \quad b = 1.5\,cm, \quad \beta_{01} = 100\,rad/m$

$$f_{c01} = \frac{3\times10^8}{2\times0.015} = 10^{10}\,Hz$$

$$100 = \frac{2\pi f}{3\times10^8}\sqrt{1-\left(\frac{10^{10}}{f}\right)^2} \Rightarrow f = 1.108\times10^{10}\,Hz$$

a)

From Eq. (10.62) for TE_{01},

$$\tilde{H}_x = 0, \quad \tilde{E}_y = 0, \quad \tilde{E}_z = 0$$

$$\tilde{E}_x = j\frac{\omega\mu b}{\pi}\hat{H}_{zm}\sin\left(\frac{\pi}{b}y\right)e^{-j\beta_{01}z}$$

or

$$\tilde{E}_x = \hat{E}_{xm}\sin\left(\frac{\pi}{b}y\right)e^{-j\beta_{01}z}$$

$$\therefore \quad \hat{E}_{xm} = j\frac{\omega\mu b}{\pi}\hat{H}_{zm}$$

$$500 = j\frac{2\pi\times1.108\times10^{10}\times4\pi\times10^{-7}\times1.5\times10^{-2}\,\hat{H}_{zm}}{\pi}$$

$$\hat{H}_{zm} = -j1.197\ A/m$$

$$\tilde{E}_x = 500\sin\left(\frac{\pi}{1.5\times10^{-2}}y\right)e^{-j100z} = 500\sin(209.43y)\,e^{-j100z}$$

$$E_x(t) = 500\sin(209.43y)\cos(2\pi\times1.108\times10^{10}t - 100z)$$

$$\tilde{H}_y = j\frac{100\times1.5\times10^{-2}}{\pi}(-j1.197)\sin(209.43y)\,e^{-j100z}$$

$$\tilde{H}_y = 0.5725 \sin(209.43\, y)\, e^{-j100z}$$

$$H_y(t) = 0.572 \sin(209.43y) \cos(2\pi \times 1.108 \times 10^{10} t - 100z)$$

$$\tilde{H}_z = -j\,1.197 \cos(209.43y)\, e^{-j100z}$$

$$H_z(t) = 1.197 \cos(209.43y) \cos\left(2\pi \times 1.108 \times 10^{10} t - 100z - \frac{\pi}{2}\right)$$

b) $$\eta_{01}^{TE} = \frac{377}{\sqrt{1-\left(\frac{10^{10}}{1.108\times10^{10}}\right)^2}} = 875.45\,\Omega$$

From Eq. (10.72)

$$\langle P_{01} \rangle = 875.45 \left[\frac{(100)^2 \times (10^{-2})^3 (1.5\times10^{-2})^3 \times 1.197^2}{8\pi^2 (10^{-2})^2} \right]$$

$$\langle P_{01} \rangle = 5.36 \times 10^{-3}\ W \quad \text{or} \quad 5.36\ mW$$

Exercise 10.6

$a = 2\,cm$, $b = 1\,cm$, TE_{10}, $f = 9\,GHz$, $E = 20\,V/cm$ at $z = 0$

a) $$f_{c10} = \frac{3\times10^8}{2\times0.02} = 7.5\times10^9\ Hz \quad \text{or} \quad 7.5\,GHz$$

b) $$\beta_{10} = \frac{2\pi \times 9\times10^9}{3\times10^8} \sqrt{1-\left(\frac{7.5}{9}\right)^2} = 104.19\ rad/m$$

$$\hat{\gamma}_{10} = j\,104.19$$

c) Phase velocity: $$u_{p10} = \frac{3\times10^8}{\sqrt{1-\left(\frac{7.5}{9}\right)^2}} = 5.43\times10^8\ m/s$$

Group velocity: $$u_{g10} = 3\times10^8 \sqrt{1-\left(\frac{7.5}{9}\right)^2} = 1.66\times10^8\ m/s$$

d) $\eta_{10}^{TE} = \dfrac{377}{\sqrt{1-(\frac{7.5}{9})^2}} = 682.02\ \Omega$

e) $\langle P_{10} \rangle = 682.02\ \dfrac{104.19^2 \times (2\times10^{-2})^3 \times (10^{-2})^3\ 4.42^2}{8\pi^2 \times (10^{-2})^2} = 0.147\ W$

$H_{2m} = \dfrac{20\times10^2\pi}{2\pi\times9\times10^9\times4\pi\times10^{-7}\times2\times10^{-2}}$

$H_{2m} = 4.42\ A/m$

Exercise 10.10

$a = 2\,cm,\ b = 1\,cm,\ c = 4\,cm,\ TM_{101}$

$f_{101} = \dfrac{1}{2\sqrt{\mu_0\epsilon_0}}\sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.04}\right)^2} = 8.385\times10^9\ Hz$

$\delta_c = \dfrac{1}{\sqrt{5.8\times10^7 \times 4\pi\times10^{-7}\times 8.385\times10^9\,\pi}} = 7.217\times10^{-7}\ m$

From Eq. (10.124)

$Q = 7299$

From Eq. (10.123)

$P_d = 4.539\times10^{-5} H_{2m}^2\ \ W$

Problem 10.14

$a = 3\,cm,\ b = 1\,cm,\ l = 1\,m,\ f = 12\,GHz,\ TE_{10}$

$\tan\delta = 10^{-4},\ \sigma = 5.8\times10^7\ S/m$

$f_{c10} = \dfrac{3\times10^8}{2\times0.03} = 5\times10^9\ Hz$

$$\delta_c = \frac{1}{\sqrt{5.8\times10^7 \times 4\pi\times10^{-7}\times 12\times10^9\pi}} = 6.03\times10^{-7}\ m$$

$$\sigma_d = \omega\epsilon\tan\delta = 2\pi\times12\times10^9\times 8.85\times10^{-12}\times10^{-4} = 6.67\times10^{-5}\ S/m$$

$$\alpha_{c_{10}} = \frac{1+\frac{2\times0.01}{0.03}\left(\frac{5}{12}\right)^2}{5.8\times10^7\times6.03\times10^{-7}\times377\times0.01\sqrt{1-\left(\frac{5}{12}\right)^2}} = 9.308\times10^{-3}\ Np/m$$

$$\alpha_{d_{10}} = \frac{6.67\times10^{-5}}{2}\times377\sqrt{1-\left(\frac{5}{12}\right)^2} = 1.143\times10^{-2}\ Np/m$$

Problem 10.16

$l = 10m$, $a = 4cm$, $b = 3cm$, TE_{10} , $f = 4GHz$, $E = 1000V/m$

$\tan\delta = 10^{-4}$, $\sigma = 5.8\times10^7\ S/m$

$$f_{c_{10}} = \frac{3\times10^8}{2\times0.04} = 3.75\times10^9\ Hz \qquad f_{c_{10}} = 3.75\ GHz$$

$$\delta_c = \frac{1}{\sqrt{5.8\times10^7\times4\pi\times10^{-7}\times4\times10^9\pi}} = 1.045\times10^{-6}\ m$$

$$\alpha_{c_{10}} = \frac{1}{5.8\times10^7\times1.045\times10^{-6}\times377\times0.03}\ \frac{\left[1+\frac{2\times0.03}{0.04}\left(\frac{3.75}{4}\right)^2\right]}{\sqrt{1-\left(\frac{3.75}{4}\right)^2}}$$

$$\alpha_{c_{10}} = 9.718\times10^{-3}\ Np/m$$

$$\sigma_d = \omega\epsilon\tan\delta = 2\pi\times4\times10^9\times8.85\times10^{-12}\times10^{-4}$$

$$\sigma_d = 2.224\times10^{-5}\ S/m$$

$$\alpha_{d_{10}} = \frac{2.224\times10^{-5}}{2}\times377\sqrt{1-\left(\frac{3.75}{4}\right)^2} = 1.459\times10^{-3}\ Np/m$$

$$\alpha_{10} = \alpha_{c_{10}} + \alpha_{d_{10}} = 9.718\times10^{-3} + 1.459\times10^{-3} = 11.18\times10^{-3}\ Np/m$$

$$H_{2m} = \frac{1000\,\pi}{2\pi\times4\times10^{9}\times4\pi\times10^{-7}\times0.04} = 2.49\ A/m$$

$$\beta_{10} = \frac{2\pi\times4\times10^{9}}{3\times10^{8}}\sqrt{1-\left(\frac{3.75}{4}\right)^{2}} = 29.15\ rad/m$$

$$\langle P_{10}\rangle_{z} = \frac{2\pi\times4\times10^{9}\times4\pi\times10^{-7}\times0.04^{3}\times0.03}{4\pi^{2}}\,29.15\times2.49^{2}\,e^{-2\times11.18\times10^{-3}\times10}$$

$$\langle P_{10}\rangle_{z} = 0.222\ W$$

Problem 10.18

$a = 5\,cm,\quad b = 2\,cm,\quad l = 7\,cm,\quad TM_{110}$

$$f_{110} = \frac{1}{2\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}}\sqrt{\left(\frac{1}{0.05}\right)^{2}+\left(\frac{1}{0.02}\right)^{2}}$$

$$f_{110} = 8.07\times10^{9}\ Hz \quad \text{or} \quad 8.07\ GHz.$$