EE351 - Spring 2006 - HW7 Solutions

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Exercise 11.5

Verify equations (11.35) and (11.36)

$\widetilde{A} = \frac{\mu \widetilde{I} \ell}{4\pi r} \mathrm{e}^{-\jmath\beta r} \left[\cos\theta \hat{a_r} - \sin\theta \hat{a_\theta}\right]$
$\widetilde{B} = \nabla \times \widetilde{A} \Rightarrow \widetilde{H} = \jmath \frac{\beta \widetilde{I} \ell}{4\pi r} \left(1 + \frac{1}{\jmath\beta r}\right) \sin\theta \mathrm{e}^{-\jmath\beta r} \hat{a_\phi}$
$\widetilde{E} = \frac{1}{\jmath\omega\epsilon} \left[\nabla \times \widetilde{H}\right]$

$$\begin{aligned} \widetilde{E} &= \frac{\beta \widetilde{I} \ell}{4\pi\omega\epsilon r^2} \left(1 + \frac{1}{\jmath\beta r}\right) (2\cos\theta)\, \mathrm{e}^{-\jmath\beta r} \hat{a_r} + \\ & \frac{\jmath\beta \widetilde{I} \ell}{4\pi\omega\epsilon r} \beta \left[1 + \frac{1}{\jmath\beta r} - \frac{1}{\beta^2 r^2}\right] (\sin\theta)\, \mathrm{e}^{-\jmath\beta r} \hat{a_\theta} \end{aligned}$$

substituting $\frac{\beta}{\omega\epsilon} = \eta$ we obtain the desired equation.

Exercise 11.2

Express (11.15) as a set of three scalar equations

$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = -\mu J_x$
$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} = -\mu J_y$
$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = -\mu J_z$

Problem 11.6

Determine the radiation resistance of a short antenna if its length is $0.1\lambda_0$, where λ_0 is the wavelength in free space. If the antenna is designed to radiate 500 W, calculate the maximum value of the antenna current.
$R_{rad} = 80\pi^2 (0.1)^2 = \mathbf{7.9\Omega = R_{rad}}$
$P_{rad} = 500W$
$P_{rad} = \frac{1}{2} I^2 R_{rad} \Rightarrow I = \sqrt{\frac{2P_{rad}}{R_{rad}}} = \sqrt{\frac{2\times 500}{7.9}} = \mathbf{11.25A = I}$

Problem 11.7

A short antenna causes a maximum field intensity of 6 mV/m at a distance of 10 km. Write the expressions for the fields and calculate the total power radiated by the antenna.
From (11.42), $\frac{\beta I \ell \eta}{4\pi} = 6 \times 10^{-3} \times 10 \times 10^3 = 60; r = 10km; \eta = 120\pi\Omega$
$\widetilde{E} = \jmath \frac{60}{r} \sin\theta \mathrm{e}^{-\jmath\beta r}$
$\widetilde{H} = \jmath \frac{1}{2\pi r} \sin\theta \mathrm{e}^{-\jmath\beta r}$
$\left\langle \hat{S}_r \right\rangle = \frac{1}{2} \left[\widetilde{E} \times \widetilde{H}^* \right] \hat{a_r} = \frac{15}{\pi r^2} \sin^2\theta W/m^2$
$P_r = \frac{15}{r} \int_0^{\pi} \sin^3\theta d\theta \int_0^{2\pi} d\phi = 40W$

Problem 11.11

A quarter-wave monopole antenna is mounted on a reflecting plane. Write expressions for the fields, average power density, total radiated power, and radiation resistance. What is the radiation resistance in free space.

$\widetilde{E}_\theta$ is given in (11.77), $\widetilde{H}_\theta$ in (11.76b) and $\left\langle \hat{S} \right\rangle$ in (11.78). These equations are true for a $\frac{\lambda}{4}$ monopole above $z \geq 0$ plane. However, power radiated will be $\frac{1}{2}$ of that dipole. Hence:

$P_{rad}|_{mono} = \frac{1.219}{8\pi} \eta I_0^2$
$R_{rad} = \frac{1.219}{8\pi} \eta = 36.57\Omega$ in free space