

## Solution to homework # 9

Problem 1:

Discrete forms of Laplace equation in 1D:

$$\frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{h^2} = 0$$

$$\phi_i = \frac{1}{2}(\phi_{i-1} + \phi_{i+1})$$

Assume initially: all potentials at interior points are set to zeros

At end of iteration 1:

$$\phi_1 = \frac{1}{2}(10 + 0) = 5.0V$$

$$\phi_2 = \frac{1}{2}(5 + 0) = 2.5V$$

$$\phi_3 = \frac{1}{2}(2.5 + 0) = 1.25V$$

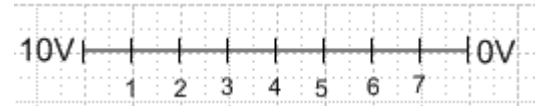
$$\phi_4 = \frac{1}{2}(1.25 + 0) = 0.625V$$

$$\phi_5 = \frac{1}{2}(0.625 + 0) = 0.3125V$$

$$\phi_6 = \frac{1}{2}(0.3125 + 0) = 0.15625V$$

$$\phi_7 = \frac{1}{2}(0.15625 + 0) = 0.078125V$$

After three iterations  $\phi_3 = 2.89V$ .....,  $\phi_5 = 1.094V$



B. Converged solution at interior points is:

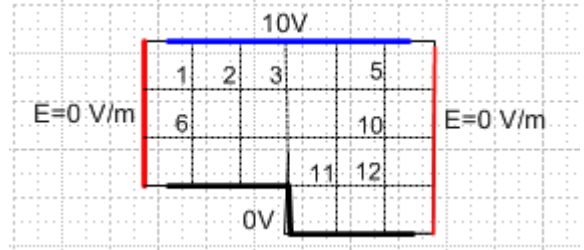
|                   |   |          |   |
|-------------------|---|----------|---|
| potential at node | 1 | 8.745562 | V |
| potential at node | 2 | 7.492424 | V |
| potential at node | 3 | 6.240855 | V |
| potential at node | 4 | 4.990855 | V |
| potential at node | 5 | 3.742194 | V |
| potential at node | 6 | 2.494481 | V |
| potential at node | 7 | 1.247240 | V |

C. Matrix equation is:

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**Problem 2:**

Solve the following equations iteratively at each node:



Initially set all potentials to zeros.

$$\begin{aligned} \phi_1 &= \frac{1}{3} * (10 + \phi_6 + \phi_2) \\ \phi_2 &= 0.25 * (\phi_1 + 10 + \phi_7 + \phi_3) \\ \phi_3 &= 0.25 * (\phi_2 + 10 + \phi_8 + \phi_4) \\ \phi_4 &= 0.25 * (\phi_3 + 10 + \phi_9 + \phi_5) \\ \phi_5 &= \frac{1}{3} * (\phi_4 + 10 + \phi_{10}) \\ \phi_6 &= \frac{1}{3} * (\phi_1 + \phi_7) \\ \phi_7 &= 0.25 * (\phi_6 + \phi_2 + \phi_8) \\ \phi_8 &= 0.25 * (\phi_7 + \phi_3 + \phi_9) \\ \phi_9 &= 0.25 * (\phi_8 + \phi_4 + \phi_{11} + \phi_{10}) \\ \phi_{10} &= \frac{1}{3} * (\phi_9 + \phi_5 + \phi_{12}) \\ \phi_{11} &= 0.25 * (\phi_9 + \phi_{12}) \\ \phi_{12} &= \frac{1}{3} * (\phi_{11} + \phi_{10}) \end{aligned}$$

At end of iteration 2:

|                   |     |           |   |
|-------------------|-----|-----------|---|
| potential at node | 1=  | 4.062500  | V |
| potential at node | 2=  | 4.570312  | V |
| potential at node | 3=  | 4.736328  | V |
| potential at node | 4=  | 4.790039  | V |
| potential at node | 5=  | 4.806519  | V |
| potential at node | 6=  | 1.406250  | V |
| potential at node | 7=  | 1.757812  | V |
| potential at node | 8=  | 1.896973  | V |
| potential at node | 9=  | 2.016602  | V |
| potential at node | 10= | 2.068481  | V |
| potential at node | 11= | 0.5903625 | V |
| potential at node | 12= | 0.7509232 | V |

The matrix equation is as follows:

$$\begin{bmatrix} -3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & & & 0 & 1 & & & & & 0 \\ 0 & 1 & -4 & 1 & & 0 & 1 & & & & & 0 \\ 0 & & 1 & -4 & 1 & & 0 & 1 & & & & 0 \\ 0 & & & 1 & -3 & & 0 & 1 & 0 & & & 0 \\ 1 & 0 & & & -3 & 1 & & 0 & 1 & 0 & & 0 \\ 0 & 1 & 0 & & 1 & -4 & 1 & & 0 & 1 & & 0 \\ 0 & & 1 & 0 & & 1 & -4 & 1 & & 0 & & 0 \\ 0 & & & 1 & 0 & & 1 & -4 & 1 & 0 & & 0 \\ 0 & & & & 1 & 0 & & 1 & -3 & 1 & & 0 \\ 0 & & & & & 0 & 1 & 4 & 1 & \phi_1 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} -10 \\ -10 \\ -10 \\ -10 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

|                   |    |          |   |
|-------------------|----|----------|---|
| potential at node | 1  | 6.710033 | V |
| potential at node | 2  | 6.748701 | V |
| potential at node | 3  | 6.849855 | V |
| potential at node | 4  | 7.040864 | V |
| potential at node | 5  | 7.158413 | V |
| potential at node | 6  | 3.381655 | V |
| potential at node | 7  | 3.435083 | V |
| potential at node | 8  | 3.610065 | V |
| potential at node | 9  | 4.155449 | V |
| potential at node | 10 | 4.434654 | V |
| potential at node | 11 | 1.536425 | V |
| potential at node | 12 | 1.990332 | V |

Problem 3: see example in the handout.