

Homework # 9 Solution: Problem 3:

Solve the following system of nonlinear equations using Newton's method.

As an initial guess assume  $x_0 = 1, y_0 = 1$

$$x^2 y - y^2 = 3$$

$$2xy^2 - x^3 y = 12$$

(A) Set up the Jacobian matrix

(B) What is the value of  $x$  and  $y$  at the end iteration 2.

(C) Write a Matlab or C code and determine the values of  $x$  and  $y$  assuming a convergence criterion  $\varepsilon = 0.0001$

Solution:

1. Define the functions  $F(x,y)$  and  $G(x,y)$

$$F(x, y) = x^2 y - y^2 - 3 = 0$$

$$G(x, y) = 2xy^2 - x^3 y - 12 = 0$$

2. Compute the derivatives:

$$F'_x(x, y) = 2xy$$

$$F'_y(x, y) = x^2 - 2y$$

$$G'_x(x, y) = 2y^2 - 3x^2 y$$

$$G'_y(x, y) = 4xy - x^3$$

3. Determine the values and Jacobian matrix at  $x = x_0 = 1; y = y_0 = 1$

$$F(x, y) = -3$$

$$G(x, y) = -11$$

$$F'_x(x, y) = 2$$

$$F'_y(x, y) = -1$$

$$G'_x(x, y) = -1$$

$$G'_y(x, y) = 3$$

$$J = \begin{bmatrix} 2xy & x^2 - 2y \\ 2y^2 - 3x^2 y & 4xy - x^3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

4. Determine  $\delta x$  and  $\delta y$ :

$$\delta x = -\frac{\begin{vmatrix} -3 & -1 \\ -11 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}} = -\frac{-20}{5} = 4; \delta y = -\frac{\begin{vmatrix} 2 & -3 \\ -1 & -11 \end{vmatrix}}{5} = -\frac{-25}{5} = 5$$

$$x_1 = x_0 + \delta x = 1 + 4 = 5$$

$$y_1 = y_0 + \delta y = 1 + 5 = 6$$

5. Check convergence:

Maximum change  $\Delta = |\delta y| = 5 > 0.0001$ , solution not converged. Repeat steps 3 through 5

For iteration 2, use  $x = x_1 = 5; y = y_1 = 6$  in step 3 and repeat calculations.