Problem 1: Security Assessment. Consider the following 3-bus power system with values given in per unit.

Transmission lines are purely inductive where \( B_{ij} = 10 \) \((i \neq j)\). Ignore shunt capacitances. A fast-decoupled power flow solution shows that \( V_3 = 0.95 \) p.u., the voltage angles are \( \theta_2 = -3^\circ \), \( \theta_3 = -10^\circ \), and the line currents are \( I_{12} = 0.733 \angle 41.47^\circ \) p.u., \( I_{23} = 1.577 \angle -45.77^\circ \) p.u. Suppose the voltage \( V_3 \) has to satisfy \( 0.95 \leq V_3 \leq 1.05 \) p.u. and the line currents should be limited to within 2.05 p.u.

A) Is the system operating in the normal, emergency, or restorative state? Justify your answer.

Solution: The currents are less than 2.05 pu and \( V_3 \) is within its limits. The system is operating in the normal state.
B) Due to the weather condition, the dispatcher is concerned about the impact of the potential outage of the line between bus 2 and 3.

With the outage of line (2, 3), the operating conditions after one iteration of the fast-decoupled power flow is \( \theta_2 = 3.43^\circ, V_3 = 0.878 \angle -16.42^\circ \) p.u.,

\( I_{12} = 0.79 \angle 142.55^\circ \) p.u., and \( I_{13} = 2.94 \angle -147.55^\circ \) p.u.

Is the system in a secure or insecure state? Justify your answer.

Solution: \(|I_{13}| = 2.94 > 2.05, V_3 = 0.878 < 0.95\)

The system is insecure.
C) Now a new on-line load forecast shows that the load at bus 3 is going to decline to the level of $S_{D3} = 2 + j0.5$ p.u. in the next 30 minutes.

Under this situation, the operating conditions are $	heta = 3.43^\circ$, $V_3 = 0.95\angle -11.46^\circ$ p.u., $I_{12} = 0.79\angle 142.55^\circ$ p.u., and $I_{13} = 2.00\angle -20.06^\circ$ p.u.

Is the system secure or insecure for this forecasted change of load with the possible outage of line (2, 3)?

Solution: The system is secure. The currents are now all within the 2.05 pu limit and $V_3$ is within its limits.
Problem 2: System Security Concepts. The one-line diagram of a power system is shown below.

A) In on-line operation, the power system condition is given below:

<table>
<thead>
<tr>
<th></th>
<th>Voltage</th>
<th>Line Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (KV)</td>
<td>Angle (deg)</td>
</tr>
<tr>
<td>Bus 1</td>
<td>220</td>
<td>0</td>
</tr>
<tr>
<td>Bus 2</td>
<td>220</td>
<td>-0.196</td>
</tr>
<tr>
<td>Bus 3</td>
<td>220</td>
<td>-1.163</td>
</tr>
</tbody>
</table>

The system is in a **normal** state. Why? (Explain)

B) Suppose that the next contingency list contains a single contingency, i.e, line from 1 to 3 out of service. A DC power flow solution taking into account the single contingency is given as follows:

<table>
<thead>
<tr>
<th></th>
<th>Voltage</th>
<th>Line Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude (KV)</td>
<td>Angle (deg)</td>
</tr>
<tr>
<td>Bus 1</td>
<td>220</td>
<td>0</td>
</tr>
<tr>
<td>Bus 2</td>
<td>220</td>
<td>-3.222</td>
</tr>
<tr>
<td>Bus 3</td>
<td>220</td>
<td>-5.309</td>
</tr>
</tbody>
</table>

The system is now operating in a **insecure** state. Why? (Explain)
Problem 3: State Estimation. The one-line diagram of a power system is shown below.

All transmission lines have the same reactance $X = j0.1pu$, and the standard deviation of all meters is given by $\sigma = 0.01$. Assume that the state vector is $(\theta_2 \quad \theta_3 \quad \theta_4)^T$ and $\theta_1 = 0$.

A) Write down the measurement matrix $H$.

\[
\begin{align*}
P_2 &= \frac{\theta_2 - \theta_1}{X_{21}} + \frac{\theta_2 - \theta_4}{X_{24}} = \left(\frac{1}{X_{21}} + \frac{1}{X_{24}}\right)\theta_2 + \left(-\frac{1}{X_{24}}\right)\theta_4 \\
P_{34} &= \frac{\theta_3 - \theta_4}{X_{34}} \\
P_{43} &= \frac{\theta_4 - \theta_3}{X_{34}} \\
H &= \begin{pmatrix} 1/X_{21} + 1/X_{24} & 0 & -1/X_{24} \\ 0 & 1/X_{34} & -1/X_{34} \\ 0 & -1/X_{34} & 1/X_{34} \end{pmatrix} = \begin{pmatrix} 20 & 0 & -10 \\ 0 & 10 & -10 \\ 0 & -10 & 10 \end{pmatrix}
\end{align*}
\]

B) Show why the system is unobservable.

\[
R = \begin{pmatrix} 0.01^2 & 0 & 0 \\ 0 & 0.01^2 & 0 \\ 0 & 0 & 0.01^2 \end{pmatrix}
\]
\[
H^T R^{-1}H = \begin{pmatrix}
4000000 & 0 & -2000000 \\
0 & 2000000 & -2000000 \\
-2000000 & -2000000 & 3000000
\end{pmatrix}
\]
Note that \( H \) has rank 2

This matrix is singular, i.e. \( \det(H^T R^{-1}H) = 0 \), thus the system is unobservable

C) Move the location of \( \textbf{M3} \) to the line 4 – 2 as shown below. Show why the new system is observable.

\[
H = \begin{pmatrix}
1/X_{21} + 1/X_{24} & 0 & -1/X_{24} \\
0 & 1/X_{34} & -1/X_{34} \\
-1/X_{42} & 0 & 1/X_{42}
\end{pmatrix} = \begin{pmatrix}
20 & 0 & -10 \\
0 & 10 & -10 \\
-10 & 0 & 10
\end{pmatrix}
\]
Note that \( H \) has rank 3

\[
H^T R^{-1}H = \begin{pmatrix}
5000000 & 0 & -3000000 \\
0 & 1000000 & -1000000 \\
-3000000 & -1000000 & 3000000
\end{pmatrix}
\]
This matrix is not singular, i.e. \( \det(H^T R^{-1}H) = 1000 \), thus the system is observable

D) Calculate the best estimates of bus angles. Assume that \( Z \text{ measure} \) is given by:

\[
Z_{\text{measure}} = \begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3
\end{pmatrix} = \begin{pmatrix}
0.210 \\
-0.028 \\
-0.180
\end{pmatrix} \text{ pu}
\]

\[
\hat{\theta} = \begin{pmatrix}
\hat{\theta}_2 \\
\hat{\theta}_3
\end{pmatrix} = (H^T R^{-1}H)^{-1}H^T R^{-1}Z_{\text{measure}} = \begin{pmatrix}
-0.0030 \\
-0.0178 \\
-0.0150
\end{pmatrix}
\]
Problem 4: State Estimation. A Three bus system with 4 meters are shown below.

![Diagram of a three bus system with four meters showing reference bus and connections with bus angles and impedances labeled.]

The meter reading values and meter standard deviations are given as follows:

\[
\begin{align*}
M_{12} &= 64 \text{ MW} & \sigma_{12} &= 0.02 \\
M_{21} &= -59 \text{ MW} & \sigma_{21} &= 0.01 \\
M_{23} &= -38 \text{ MW} & \sigma_{23} &= 0.01 \\
M_{31} &= -4 \text{ MW} & \sigma_{31} &= 0.001
\end{align*}
\]

Calculate the best estimates of the bus angles. (Use 100 MVA as the base value for per unit.)

Solutions:

\[
\begin{align*}
Z_{12} &= M_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = 5\theta_1 - 5\theta_2 \\
Z_{21} &= M_{21} = \frac{\theta_2 - \theta_1}{X_{12}} = 5\theta_2 - 5\theta_1 \\
Z_{23} &= M_{23} = \frac{\theta_2 - \theta_3}{X_{23}} = 4\theta_2 \\
Z_{31} &= M_{31} = \frac{\theta_3 - \theta_1}{X_{31}} = -2.5\theta_1
\end{align*}
\]

\[
\begin{pmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4
\end{pmatrix} =
\begin{pmatrix}
5 & -5 \\
-5 & 5 \\
0 & 4 \\
-2.5 & 0
\end{pmatrix}
\begin{pmatrix}
\theta_1 \\
\theta_2
\end{pmatrix} =
\begin{pmatrix}
5 & -5 \\
-5 & 5 \\
0 & 4 \\
-2.5 & 0
\end{pmatrix}
\]
\[
R = \begin{pmatrix}
0.02^2 & 0 & 0 & 0 \\
0 & 0.01^2 & 0 & 0 \\
0 & 0 & 0.01^2 & 0 \\
0 & 0 & 0 & 0.001^2
\end{pmatrix}
\]

\[
Z_{\text{measure}} = \begin{pmatrix}
0.64 \\
-0.59 \\
-0.38 \\
-0.04
\end{pmatrix}
\]

\[
\hat{\theta} = \begin{pmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2
\end{pmatrix} = (H^T R^{-1} H)^{-1} H^T R^{-1} Z_{\text{measure}} = \begin{pmatrix}
0.0161 \\
-0.1009
\end{pmatrix}
\]
Problem 5: Observability. The following system is an unobservable one. There are five meters in total.

Move the location of one meter to make the power system observable and draw the five meter locations after you move the meter you choose. Show why the new system is observable.
Solutions:

\[ Z_{15} = B_{15} (\theta_1 - \theta_5) \]
\[ Z_{12} = B_{12} (\theta_1 - \theta_2) \]
\[ Z_{21} = B_{21} (\theta_2 - \theta_1) \]
\[ Z_{32} = B_{32} (\theta_3 - \theta_2) \]
\[ Z_{45} = B_{45} (\theta_4 - \theta_5) \]

\[
H = \begin{pmatrix}
B_{15} & 0 & 0 & 0 & -B_{15} \\
B_{12} & -B_{12} & 0 & 0 & 0 \\
-B_{21} & B_{21} & 0 & 0 & 0 \\
0 & -B_{32} & B_{32} & 0 & 0 \\
0 & 0 & 0 & B_{45} & -B_{45}
\end{pmatrix}
\]

Rank \((H\) with 1 column deleted) = 4 : Full Rank! The system is \textbf{observable}
Problem 6: Use MATLAB to solve the following problem (Please attach the MATLAB source code)

Given the network shown in below, the network is to be modeled with a DC power flow with line reactances as follows (assume 100-MVA base):

\[ x_{12} = 0.1 \text{ pu} \]
\[ x_{23} = 0.25 \text{ pu} \]

The meters are all of the same type with a standard deviation of \( \sigma = 0.01 \) pu for each. The measured values are:

\[ M_{21} = 148 \text{ MW} \]
\[ M_2 = 49 \text{ MW} \]
\[ M_{23} = -135 \text{ MW} \]
\[ M_{32} = 98 \text{ MW} \]
\[ M_3 = 105 \text{ MW} \]

A) Find the phase angle which result in a best fit to the measured values.
B) Find the value of the residual function J.
C) Is there any poor measurement? Explain which measurement may be poor and why. Remove the poor measurement and reduce the value of residual.

Solutions:

In this problem the state variables are \( \theta_2 \) and \( \theta_3 \)

\[ M_{21} = f_{21} = \frac{1}{0.1} (\theta_2 - \theta_1) = 10\theta_2 - 10\theta_1 \]
\[
M_2 = f_2 = \frac{1}{0.1}(\theta_2 - \theta_1) + \frac{1}{0.25}(\theta_2 - \theta_3) = -10\theta_1 + 14\theta_2
\]

\[
M_{23} = f_{23} = \frac{1}{0.25}(\theta_2 - \theta_3) = 4\theta_2
\]

\[
M_{32} = f_{32} = \frac{1}{0.25}(\theta_3 - \theta_2) = -4\theta_2
\]

\[
M_3 = f_3 = \frac{1}{0.25}(\theta_3 - \theta_2) = -4\theta_2
\]

The H matrix is:
\[
H = \begin{bmatrix}
-10 & 10 \\
-10 & 14 \\
0 & 4 \\
0 & -4 \\
0 & -4 \\
\end{bmatrix};
\]

The measurement vector is:
\[
\text{meas} = [1.48 \ 0.49 \ -1.35 \ 0.98 \ 1.05]
\]

First a MATLAB program is given which shows the calculation of the estimated phase angle and them the estimated quantities \(f(x^{enf})\) When all the measurements are included, \(J\) has a value of 852.7143, it appears that the M32 measurement is bad since it is quite different from the value estimated. When M32 is removed (by setting its standard deviation to 1000) and the estimate recalculated, the value of \(J\) falls to 27.0.

```matlab
close all;
clear all;
clc;

%% Problem 6) A and B
meas=[1.48 0.49 -1.35 0.98 1.05];
H=[-10 10; -10 14; 0 4; 0 -4; 0 -4];
% H=[10 0; 14 -4; 4 -4; 4 4];
R=diag([0.01]^2 [0.01]^2 [0.01]^2 [0.01]^2 [0.01]^2]);
```
\text{thetaest} = \text{inv}(H^{*}\text{inv}(R)^{*}H)^{*}\text{inv}(R)^{*}\text{meas}^{*};

display(\text{thetaest});
\text{angles} = \text{thetaest}^{*}(180/\pi);
display(\text{angles});

fx\_est = H^{*}\text{thetaest};
fprintf('\text{n}M21 = %.4f', fx\_est(1));
fprintf('\text{n}M2 = %.4f', fx\_est(2));
fprintf('\text{n}M23 = %.4f', fx\_est(3));
fprintf('\text{n}M32 = %.4f', fx\_est(4));
fprintf('\text{n}M3 = %.4f', fx\_est(5));
fprintf('\text{n}');

J = 0.0;
\text{for } i=1:5
  J = J + ((\text{meas}(i) - \text{fx\_est}(i))^2)/R(i,i);
\text{end}
display(J);

fprintf('--------------------------------------------------------------------------------\n');

%%% Problem 6) C
\text{meas} = [1.48 0.49 -1.35 0.98 1.05];
H = [-10 10; -10 14; 0 4; 0 -4; 0 -4];
% \text{H} = [10 0; 14 -4; 4 -4; 4 -4];
R = \text{diag}([(0.01)^2 (0.01)^2 (1000)^2 (0.01)^2 (0.01)^2]);
\text{thetaest} = \text{inv}(H^{*}\text{inv}(R)^{*}H)^{*}\text{inv}(R)^{*}\text{meas}^{*};

display(\text{thetaest});
\text{angles} = \text{thetaest}^{*}(180/\pi);
display(\text{angles});

fx\_est = H^{*}\text{thetaest};
fprintf('\text{n}M21 = %.4f', fx\_est(1));
fprintf('\text{n}M2 = %.4f', fx\_est(2));
fprintf('\text{n}M23 = %.4f', fx\_est(3));
fprintf('\text{n}M32 = %.4f', fx\_est(4));
fprintf('\text{n}M3 = %.4f', fx\_est(5));
fprintf('\text{n}');

J = 0.0;
\text{for } i=1:5
  J = J + ((\text{meas}(i) - \text{fx\_est}(i))^2)/R(i,i);
\text{end}
display(J);

\textbf{Result:}

\text{thetaest} =

-0.4306
-0.2768
angles =

-24.6740
-15.8587

M21 = 1.5386
M2  = 0.4314
M23 = -1.1071
M32 = 1.1071
M3  = 1.1071

J =

852.7143

thetaest =

-0.4015
-0.2525

angles =

-23.0043
-14.4672

M21 = 1.4900
M2  = 0.4800
M23 = -1.0100
M32 = 1.0100
M3  = 1.0100
\[ J = 27.0000 \]