OPF: Optimality Conditions

For each of the following questions, circle the most appropriate answer from the 3 options. No explanation is needed.

A) The optimality condition of an **unconstrained** optimization problem \( \max_{p_1, p_2} f(p_1, p_2) \) if the optimal solution is given by \( p^* \).

\[
\nabla f(p^*) = \left( \frac{\partial f}{\partial p_1} (p^*) \quad \frac{\partial f}{\partial p_2} (p^*) \right)^T = 0
\]

B) Necessary optimality condition of the following **constrained** optimization problem if the optimal solution is given by \( p^* \).

\[
\max_{p_1, p_2} f(p_1, p_2) \quad \text{s.t.} \quad \begin{align*}
    h_1(p_1, p_2) &\leq 0 \\
    h_2(p_1, p_2) &\leq 0 \\
    p_1 &\geq 1, \quad p_2 \geq 1
\end{align*}
\]

\[
\nabla f(p^*) + \mu_1 \nabla h_1(p^*) + \mu_2 \nabla h_2(p^*) = 0
\]

C) Necessary optimality condition of the following **constrained** optimization problem if an equality constrain \( g(p_1, p_2) \) is included, and the optimal solution is given by \( p^* \).
\[
\max_{p_1, p_2} f(p_1, p_2)
\]
\[
s.t.
\]
\[
h_1(p_1, p_2) \leq 0
\]
\[
h_2(p_1, p_2) \leq 0
\]
\[
g(p_1, p_2) = 0
\]
\[
p_1 \geq 1, p_2 \geq 1
\]

\[
\nabla f(p^*) + \mu \nabla h_1(p^*) + \lambda \nabla g(p^*) = 0
\]