1. A passive circuit has input voltage \( v_i(t) \) and output voltage \( v_o(t) \). The output voltage is across a series RC configuration, as shown below.

The capacitor voltage shown in the figure satisfies the third-order differential equation
\[
\frac{d^3v_c(t)}{dt^3} + 7\frac{d^2v_c(t)}{dt^2} + 14\frac{dv_c(t)}{dt} + 8v_c(t) = 8v_i(t) \quad \text{for} \quad t \geq 0.
\]
Assume that there is no initial energy stored in the circuit (so all initial conditions are zero).

a. (10 points) Find a state variable representation for the system,
\[
\dot{x} = Ax + Bu, \quad y = Cx + Du.
\]
Specify the A, B, C, and D matrices.

b. (10 points) Determine the transfer function for the circuit,
\[
H(s) = \frac{V_o(s)}{V_i(s)}.
\]
(Express in terms of the unspecified values \( R \) and \( C \) in the figure. Note that \( V_o(s) \) is not the same as \( V_c(s) \).)
2. The switch in the circuit below has been in position \( a \) for a long time, and at time \( t = 0 \) is moved to position \( b \). The circuit parameter values are \( R_1 = 100 \) ohms, \( R_2 = 400 \) ohms, \( L = 10 \) mH, and \( i_s(t) = 5 \) mA.

   a. (5 points) Find the initial condition current, \( i_L(0^+) \).
   b. (5 points) Find the differential equation for \( i_L(t) \), for \( t \geq 0 \).
   c. (10 points) Use Laplace transforms to solve for \( i_L(t) \) for \( t \geq 0 \).
   d. (5 points) Determine \( v_0(t) \) for \( t \geq 0 \).

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![Circuit Diagram]

\( i_s(t) = 5 \) mA

\( t = 0 \)

\( i_L(t) \)

\( R_1 \)

\( R_2 \)

\( L \)

\( v_0(t) \)
3. A circuit with input voltage \( v_i(t) \) and output voltage \( v_o(t) \) has transfer function

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s}{s + 5}
\]

The input voltage is \( v_i(t) = 10 \ u(t) \) V, where \( u(t) \) is the unit-step function. Use the initial and final value theorems to determine

a. (8 points) Initial value: \( v_o(0^+) \).

b. (7 points) Final value: \( \lim_{t\to\infty} v_o(t) \).

c. (10 points) Use the inverse Laplace transform to find \( v_o(t) \). Verify that your solution is consistent with the results in parts a) and b).
4. Consider the circuit below with input voltage $v_i(t) = 5$ volts, and output voltage $v_o(t)$. The switch has been in position “a” for a long time, and at time $t = 0$ is moved to position “b”. The circuit parameter values are $R_1 = 2$ ohm, $R_2 = 3$ ohms, $C = 0.5$ F, and $L = 1$ H.

![Circuit Diagram]

a. (5 points) Determine the initial conditions for the circuit (initial capacitor voltage and initial inductor current).

b. (10 points) Use Laplace Transforms and complex impedances and solve for $I(s)$ (in terms of the initial conditions).

c. (5 points) Use part b) and find the output voltage in the Laplace domain, $V_o(s)$.

d. (10 points) Substitute in the specific parameter values and use the (inverse) Laplace transform to find $v_o(t)$ for $t \geq 0$. 
Useful Formulae

\[ v = L \frac{di}{dt} \quad \quad v = C \frac{dv}{dt} \quad \quad v = iR \quad \quad F = ma \quad \quad c = 2\pi r \quad \quad E = mc^2 \]

\[ L\{u(t)\} = \frac{1}{s} \quad \quad L\{tu(t)\} = \frac{1}{s^2} \quad \quad L\{e^{-at}u(t)\} = \frac{1}{s + a} \]

\[ L\{\sin(\omega t) \ u(t)\} = \frac{\omega}{s^2 + \omega^2} \quad \quad L\{\cos(\omega t) \ u(t)\} = \frac{s}{s^2 + \omega^2} \]

\[ L\{e^{-at}\sin(\omega t) \ u(t)\} = \frac{\omega}{(s + a)^2 + \omega^2} \quad \quad L\{e^{-at}\cos(\omega t) \ u(t)\} = \frac{s + a}{(s + a)^2 + \omega^2} \]

\[ L\left(\frac{dx(t)}{dt}\right) = sX(s) - x(0^-) \quad \quad L\left(\frac{d^2x(t)}{dt^2}\right) = s^2X(s) - sx(0^-) - \frac{dx(0^-)}{dt} \]

\[ \lim_{t \to \infty} x(t) = \lim_{s \to 0}sX(s) \quad \quad \lim_{t \to 0^+} x(t) = \lim_{s \to \infty}sX(s) \]

\[ \int_0^t x(t)dt \to \frac{X(s)}{s} \quad \quad e^{-at}x(t) \to X(s + a) \quad \quad x(at), \ a > 0 \to \frac{1}{a}X\left(\frac{s}{a}\right) \]

\[ tx(t) \to -\frac{dX(s)}{ds} \quad \quad x(t-a)u(t-a), \ a > 0 \to e^{-as}X(s) \]