1. Any periodic function can be represented as a sum of an even function and an odd function, i.e.,

\[ f(t) = f_e(t) + f_o(t) \]

where \( f(t) \) is an arbitrary periodic function, \( f_e(t) \) is an even function, and \( f_o(t) \) is an odd function.

Show that \( f_e(t) \) and \( f_o(t) \) are given by

\[ f_e(t) = \frac{f(t) + f(-t)}{2}, \]
\[ f_o(t) = \frac{f(t) - f(-t)}{2}. \]

2. Consider a periodic function \( f(t) \) whose first period is described by

\[ f(t) = \begin{cases} 1 & 0 \leq t < \frac{3T}{4}, \\ 0 & \frac{3T}{4} \leq t < T \end{cases} \]

(a) Make a sketch of this function over a few periods.
(b) Determine the even part of this function and make a sketch of that.
(c) Determine the odd part of this function and make a sketch of that.

3. Find the Fourier series representation of the function given in Problem 2. (Do not split it into even and odd functions.)

4. The even function you found in Problem 2b is not quarter-wave symmetric, but it can be made quarter-wave symmetric with a simple manipulation. What is that manipulation? After doing that manipulation, find the Fourier series representation of the function. Then, “undo” that manipulation to obtain the Fourier series of the even function.

5. Determine the Fourier series of the odd function you found in Problem 2c.

6. Show that the sum of the Fourier series representations of the odd and even functions you found in the last two problems agrees with the series you found in Problem 3.

7. Determine the alternative trigonometric form (described in Sec. 16.4) of the Fourier series which describes the function given in Problem 2.

8. Problem 16.26 from the text.