Using the method of images, it is trivial to find a closed-form solution for the charge distribution over an infinite ground plane when there is a point charge above the plane. However, it is not possible to find a closed-form solution when the ground plane is replaced with a finite plate. Consider a point charge of +1 nC located 1/2 m above a 1 m × 1 m plate which has a potential of zero volts. Assume the four corners of the plate are located at \((x, y, z) = (0, 0, 0), (1, 0, 0), (1, 1, 0), \) and \((0, 1, 0)\). The point charge is located directly above the center of the plate, i.e., at \((1/2, 1/2, 1/2)\). The goal is to find the charge distribution over the finite plate. We will solve this problem using the method of moments and Matlab. Before touching Matlab, make sure you do the following:

1. Write the equation which gives the potential for an observation point on the plate in terms of the charge which is present. Provide as much detail as possible. Note that we will assume the plate, although finite, is still the “ground plane” and hence it is at a potential of 0 V.

2. Assume the charge on the plate is to be modeled using 400 square pulse basis functions (20 along each side of the plate). What are the expressions for \(Z_{mn}(m \neq n)\) and \(Z_{mm}\)? Provide both exact and approximate expressions for \(Z_{mn}\) (the exact expression may be left as an integral).

3. Write an equation for the elements of the forcing vector.

Once you have obtained these equations, plugging them into Matlab should be fairly easy (provided you are familiar with Matlab!). Submit these equations together with a print-out of your Matlab code and a plot of the charge over the plate. The plot of surface charge can be a surface plot or merely a plot of the basis-function coefficients (i.e., an “x-y” plot of charge vs. basis-function number).