Extra Credit Details

For extra credit on HW 7 you were told:

Problem 13.10. You can forget the hint given by the author. All you really have to do is start from Eq. 13.16 and change the limits from $\pm \lambda/4$ to $\pm l/2$.

Unfortunately the expression the author gives is not correct. You will obtain the expression he gives for $H_{\phi s}$ if the current is

$$I(z) = I_0 \sin[\beta((l/2) - |z|)].$$

But, of course, that is not the given current! Sigh.

If you use the current that actually pertains to this problem, here is the correct expression you should obtain for $H_{\phi s}$:

$$H_{\phi s} = \frac{j I_0 e^{j \beta r}}{2\pi r \sin(\theta)} \left[ \sin \left( \frac{\beta l}{2} \right) \sin \left( \frac{\beta l}{2} \cos(\theta) \right) - \cos(\theta) \cos \left( \frac{\beta l}{2} \right) \sin \left( \frac{\beta l}{2} \cos(\theta) \right) \right].$$

Note that $H_{\phi s}$ can be obtained almost directly from $A_{zs}$. We can obtain $A_{\theta a}$ by a simple vector projection (which is given in the book):

$$A_{\theta s} = -A_{zs} \sin(\theta).$$

We know in the far field that $E_{\theta s} = -j \omega A_{\theta s}$. For this particular antenna we know that there is no $\phi$ component of $A$ (and thus no $\phi$ component of $E$). In the far field we also know that the direction of propagation is given by $\mathbf{E} \times \mathbf{H}$ and this must be in the $r$ direction (radially away from the source). Thus there must be a $\phi$ component of $\mathbf{H}$.

We also know that in the far field $E$ and $H$ are related by the characteristic impedance. Putting this information together we can write:

$$H_{\phi s} = E_{\phi s}/\eta = -j \omega A_{\phi s}/\eta = j \omega \sin(\theta) A_{zs}/\eta.$$

So, for this problem, you can follow the original instructions. Obtain $A_{zs}$ from Eq. 13.16 with the appropriate limits. From that you can obtain $H_{\phi s}$ using the conversion directly above. Doing this, you should end up with the expression given above for $H_{\phi s}$ (not the one in the book!).

You will get full credit if you obtain this expression—no need to do part (b).