EE 351 — Quiz #3

NAME: ANSWER KEY

Consider a square loop with sides of 2 cm that is centered about the $z$ axis and lies in the $z = 0$ plane. The orientation of the loop is such that the sides are parallel to either the $x$ or the $y$ axis. Determine the EMF induced in the loop for the following $B$ fields:

1. $B = \hat{a}_z \cos(\omega t)$ Wb/m$^2$

The loop itself never moves. Thus to calculate the EMF for this problem (and all the rest) we can use

$$V_{emf} = -\int_S \frac{\partial B}{\partial t} \cdot ds = -\int_{x=-0.01}^{0.01} \int_{y=-0.01}^{0.01} \frac{\partial B}{\partial t} \cdot \hat{a}_z \, dx \, dy.$$ 

For this particular $B$ we have, $\frac{\partial B}{\partial t} = -\omega \sin(\omega t)\hat{a}_z$. Note that this is independent of position. Thus we obtain

$$V_{emf} = \int_S \omega \sin(\omega t)\hat{a}_z \cdot \hat{a}_z \, dx \, dy = \omega \sin(\omega t) (\text{area}) = 0.0004 \omega \sin(\omega t) \quad \text{V}$$

2. $B = \hat{a}_x 5t$ Wb/m$^2$

Note that in this case the $B$ field is oriented in the $x$ direction but the surface-normal for the loop is oriented in the $z$ direction. Thus the dot product of $B$ and $ds$ is zero and we obtain

$$V_{emf} = 0$$

3. $B = \hat{a}_z x^2 y^2$ Wb/m$^2$

Since this field is independent of time, $\frac{\partial B}{\partial t} = 0$. Thus we obtain

$$V_{emf} = 0$$

(over)
4. \( B = \hat{a}_z 10x_t \text{ Wb/m}^2 \)

In this case \( \frac{\partial B}{\partial t} = 10x \hat{a}_z \). Plugging this into the integral we obtain

\[
V_{\text{emf}} = -\int_{y=-0.01}^{y=0.01} \int_{x=-0.01}^{x=0.01} 10x \hat{a}_z \cdot \hat{a}_z \ dx \ dy
\]

\[
= -10x^2 \bigg|_{x=-0.01}^{x=0.01} \bigg|_{y=-0.01}^{y=0.01}
\]

\[
= -10 \left[ 0.01^2 - (-0.01)^2 \right] \left[ 0.01 - (-0.01) \right]
\]

\[
= 0
\]