1) \( \text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \Gamma = \frac{Z_L-Z_0}{Z_L+Z_0} \)

- **Hertzian**: \( Z_L = 80 \pi^2 \left[ \frac{1}{20} \right]^2 = 1.9739 \Omega \Rightarrow \Gamma_1 = -0.924 \Rightarrow \text{SWR}_1 = 25.33 \)
- **Linear**: \( Z_L = 20 \pi^2 \left[ \frac{1}{20} \right]^2 = 0.4935 \Omega \Rightarrow \Gamma_2 = -0.980 \Rightarrow \text{SWR}_2 = 101.32 \)

It may be worth noting that reflection coefficients do not differ by much but the SWR's differ by a factor of four!

2) Power is proportional to the square of the field and hence to the square of the array factor. The only difference between these two arrays is the number of elements. Also, for power we want the magnitude squared of \( \text{AF} \).

At \( f = 300 \text{ MHz} \), \( \lambda = 1 \text{ m} \) \quad \( \phi = 30^\circ = \frac{\pi}{6} \)

\[ \Psi = \beta d \cos \phi + \alpha = \frac{2\pi}{d} \left( \frac{1}{4} \cos 45^\circ + \frac{1}{6} \right) = \pi \left( \frac{1}{24} + \frac{1}{6} \right) = 93.639^\circ = 1.634 \]

5 elements: \( |\text{AF}_5| = \left| \frac{\sin \left( \frac{5}{2} 93.639^\circ \right)}{\sin \left( \frac{1}{2} 93.639^\circ \right)} \right| = 1.1108 \)

4 elements: \( |\text{AF}_4| = \left| \frac{\sin \left( \frac{4}{2} 93.639^\circ \right)}{\sin \left( \frac{1}{2} 93.639^\circ \right)} \right| = 0.1737 \)

Power ratio = \( \frac{|\text{AF}_5|^2}{|\text{AF}_4|^2} = \left( \frac{1.1108}{0.1737} \right)^2 = 40.87 \)

3) \( \vec{H}_S = \frac{1}{\mathcal{N}} \nabla \times \vec{A}_S = \frac{1}{\mathcal{N}} \nabla \times (A_{z5} \cos \theta \hat{r} - A_{z5} \sin \theta \hat{\theta}) \)

\[ = \frac{1}{\mathcal{N}} \nabla \times \left( \frac{e^{-j\beta r}}{r} \cos^2 \theta \hat{r} - \frac{e^{-j\beta r}}{r} \cos \theta \sin \theta \hat{\theta} \right) A_{z5}(r, \theta) \]

Then, from expression for curl (3.56), we have

\[ \vec{H}_S = \frac{1}{\mathcal{N}} \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_{z5}) - \frac{\partial A_{z5}}{\partial \theta} \right] \hat{r} = \frac{1}{\mathcal{N}} \frac{1}{r} \left[ j \beta e^{-j\beta r} \cos \theta \sin \theta + \frac{e^{-j\beta r}}{r} 2 \cos \theta \sin \theta \right] \hat{r} \]

\[ = \frac{1}{\mathcal{N}} \frac{e^{-j\beta r}}{r} \cos \theta \sin \theta \left[ j \beta + \frac{2}{r} \right] \hat{\theta} \]
4) a) Scalars \( r \) and \( r' \) are the magnitudes (lengths) of the vectors \( \vec{F} \) and \( \vec{F}' \), respectively. Since, in general, these vectors do not point in the same direction, we cannot simply take the difference of the magnitudes.

b) Since the current just has a \( \hat{z} \) component, \( \vec{A}_s \) just has a \( \hat{z} \) component.

c) We know in the far field that \( \vec{E}_s \) is given by

\[
\vec{E}_s = -jwA_0s \hat{\Theta} - jwA_\phi s \hat{\Phi}
\]

Converting \( \vec{A}_s = A_zs \hat{z} \) to spherical components we get

\[
\vec{A}_s = A_zs \cos \Theta \hat{r} - A_zs \sin \Theta \hat{\Theta}
\]

So, there is no \( \hat{\Phi} \) component of \( \vec{A}_s \) and we ignore the \( \hat{r} \) component when finding \( \vec{E}_s \) in the far field. Thus \( \vec{E}_s \) only has a \( \hat{\Theta} \) component.