Studies in Network Partitioning Based on Topological Structure

Sandip Roy
Massachusetts Institute of Technology

I. Introduction

The analysis of large-scale electrical networks, such as power systems, poses many challenges, from the computational burden required for simulation using digital computers to simply gaining a fundamental understanding of the types of behavior such systems may exhibit. Through application of elements of graph theory and eigenanalysis, researchers have recognized that the presence of weak links in a power system can effectively isolate specific dynamic behavior to geographic areas. Typically faster dynamics occur within strongly connected regions that are separated from other areas by relatively weak connections, and slower dynamics are observed throughout the entire network or between areas. This qualitative understanding is important on its own, and it has led to development of reduced-order modeling techniques for the detailed simulation of power systems. Starting from a complete model of a system, then, it has been of considerable interest to develop algorithms that appropriately partition a system based on weak links and associated localized (presumed fast) and extensive (presumed slow) behavior.

Several researchers have developed algorithms for partitioning power systems based on a separation of slow and fast dynamics which occurs in systems with tightly connected components separated by weak interconnections [1]. The algorithms suggested in these works typically involve the selection of reference generators in the system and the association of other generators and loads to each reference. Eigenvector components corresponding to the small eigenvalues of the system are used to partition a system by identifying coherence between reference generators and other generators. An alternative approach proposed by DeMarco and Wassmer uses the sensitivity of the smallest non-zero eigenvalue to determine which cuts of weak lines will partition the system into coherent groups [2]. The method is iteratively applied to partition the system into multiple areas. It is important to emphasize that this method works directly on the topological structure of the system to identify weak connections and partitions.

Like some of the other researchers, we approach partitioning from a graphical perspective. We highlight the importance of the weighted Laplacian of the graph in understanding the behavior of three types of electrical systems defined on graphs. The structure of the eigenvectors corresponding to small eigenvalues for a graph with strongly connected subcomponents with weak interconnections is analyzed and a partitioning algorithm based on this structure is reviewed. We propose an extension to the smallest non-zero eigenvalue sensitivity method to include a set of the smallest eigenvalues as a direct approach to partitioning a system into multiple areas.

The remainder of the paper is then devoted to the application and comparison of the partitioning techniques to example networks. We formulate probabilistic models for generating realistic networks and group the nodes in the network according to the methods discussed in this paper. We simulate system dynamics on these graphs, focusing specifically on the impact of line removals on the system. The simulations demonstrate that steady-state fluctuations in line current due to line removals are largely contained within the affected subcomponent, and similarly that high frequency transients are attenuated beyond the affected region. Further analysis of the simulations suggests a different, response-based metric for partitioning.

II. Graphs and Electrical Networks

In this section we relate properties of graph theory to properties of an electrical network. On an intuitive level, one expects that the topology of a network affects its behavior: we quantify these topological effects here. Specifically we review properties of the Laplacian of a graph and then represent a few common electrical networks in terms of the Laplacian.

A. Properties of the Laplacian

A graph consists of a set of vertices and a set of edges connecting pairs of vertices. Certain weighting values may be assigned to edges and vertices, and a direction may be assigned to the edges. For our purposes, we need not be concerned with edge direction, but we do consider edge weights. We assume the graph has $N$ vertices that, for purposes of reference, are ordered from 1 to $N$. Denoting $k_{ij}$ to be the weight of an edge connecting vertex $i$ and vertex $j$ ($k_{ij} = 0$ if there is no connection), the Laplacian of a weighted graph is given by

$$
K = \begin{pmatrix}
\sum_{j=1, j \neq i}^{N} k_{ij} & -k_{i2} & \cdots \\
-k_{i1} & \sum_{j=1, j \neq i}^{N} k_{ij} & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}.
$$

(1)

The weighted Laplacian is a symmetric matrix with off-diagonal elements corresponding to the negative of the edge
weights, and diagonal elements chosen so that the rows and columns sum to zero. In this study, we assume that all edge weights are non-negative. Consequently, the Laplacian is symmetric and positive semidefinite and the eigenvalues are all real and non-negative. For purposes of discussion we order the eigenvalues by increasing value:

$$\lambda_0 \leq \lambda_1 \ldots \leq \lambda_{N-1}.$$  

An especially important property of the Laplacian matrix concerns the rank of the matrix.

Zero Eigenvalue Property: The Laplacian has $M$ eigenvalues equal to zero, where $M$ is the number of disconnected components (connected subgraphs) of the graph.

If the graph is connected, then the Laplacian has a single eigenvalue equal to zero with a corresponding eigenvector with identical elements (typically taken to be 1). This is immediately evident since the rows (and columns) of this special matrix sum to zero. If a graph has disconnected parts, then the Laplacian of the isolated subgraphs, taken on their own, each have a zero eigenvalue. It is easily verified, then, that the Laplacian of a graph containing $M$ disconnected subgraphs has $M$ zero eigenvalues. This zero eigenvalue property, when taken in the context of electrical networks, provides the theoretical foundation for partitioning algorithms.

For a weakly connected graph such that the removal of a small number of edges will break the graph into two relatively well-connected subgraphs, the first non-zero eigenvalue, $\lambda_1$, is expected to be small. This follows from a perturbation argument; the Laplacian of the graph after the edges have been removed will now have two zero eigenvalues. The first non-zero eigenvalue is sometime referred to as the “algebraic connectivity” when it is used as a descriptive measure of connectivity. (It is also called the “Fiedler eigenvalue” in honor of Miroslav Fiedler who proved many theorems relating to graph theory.) For the purpose of partitioning connected graphs, several non-zero eigenvalues are also of interest to us. Using similar arguments, if a graph is easily broken into $M$ subgraphs, we would expect to the Laplacian to exhibit $M$ relatively small eigenvalues. We discuss this observation more in Section III.

**B. Electrical Networks**

The Laplacian matrix naturally appears in descriptions of electrical networks. We consider three such networks here. 1. The first is a network of resistors connected to a common ground through current sources and sinks. The vertices of the graph correspond to the nodes of the electrical network, excluding the ground node, and the edge weights correspond to the values of conductance for the resistors. The equation describing the node voltages relative to ground is

$$0 = -Kx + u$$  

where $x$ and $u$ represent the vectors of node voltages and injected currents respectively. Matrix $K$ is the Laplacian of the graph.

2. The second is the same as the first network with the addition of capacitors connecting each node to the ground node. This makes the system dynamic. The differential equation describing the dynamics of the node voltages is

$$C \frac{dx}{dt} = -Kx + u$$  

where $C$ is a diagonal matrix of capacitance values.

3. The third is the linearized, undamped swing equation model for a power system. The equation representing this electrical network takes the following form:

$$M \frac{d^2x}{dt^2} = -Kx + u$$

where $M$ is a diagonal matrix of generator inertias, $x$ is a vector of generator angle deviations from nominal, $u$ is a vector of mechanical power perturbations, and $K$ is a Laplacian matrix whose edge weights are given by the negative of generator incremental synchronizing torques.

From examination of (2)-(4) we see that the Laplacian appears prominently in the descriptions of the systems. The interpretation of the zero eigenvalue property carries over directly to the electrical networks. While the other eigenvalues of the Laplacian are not necessarily exhibited in the dynamics of systems, due to the capacitance and inertia matrices, they are of practical interest nonetheless. In the case of the undamped swing equation model, machine inertias (per-unit) do not typically vary over an order of magnitude, thus one may arguably gain insight into the electromechanical modes of oscillation by studying the corresponding Laplacian.

Methods for partitioning networks that are motivated by the zero eigenvalue property are reviewed in the next section.

**III. SLOW COHERENCY ANALYSIS AND PARTITIONING OF ELECTRICAL NETWORKS**

To motivate our study of the partitioning of networks, we first describe slow coherency-based partitioning from a graphical viewpoint. Then, an alternate approach for partitioning networks that is based on the sensitivity of the Fiedler eigenvalue of a graph to line conductances is summarized [1] and extended to simultaneously consider a relevant set of the smallest eigenvalues.

**A. A Graphical Approach to Slow Coherency-Based Partitioning**

The coherence of sets of generators and loads in their slow modes is well-studied, and several different partitioning algorithms based on coherency have been proposed [2], [3]. However, the bulk of the literature on slow coherency focuses on the separation of slow time scale and fast time scale dynamics in power systems, rather than the analysis and identification
of weak links in the underlying graph. Analysis of the structure of the underlying graph also buttresses the argument for partitioning based on slow coherence.

Specifically, we consider a graph of $N$ nodes divided into $M$ disconnected components with a weighted Laplacian $K$. Each of the components is assumed to be strongly connected internally. Next, we assume that the $M$ components of the graph have been connected by weak links. The structure of the first $M$ eigenvectors of this connected graph is determined to verify coherency among groups of nodes in the graph.

To determine the structure of the eigenvectors for the Laplacian of the connected graph, we first consider the eigenvectors of the Laplacian of the disconnected graph. $M$ of the eigenvalues of the disconnected graph are zero, and the corresponding $M$ eigenvectors may be chosen as any orthogonal set of basis functions for an $M$ dimensional space. It is useful to choose one of the basis functions as the vector of all ones, labeled $\mathbf{1}$. The remaining $M - 1$ basis functions must be orthogonal to $\mathbf{1}$, and further the components of each basis corresponding to each disconnected region of the graph must be identical so that the corresponding eigenvalue is zero. Any $M - 1$ orthogonal vectors in this space can be used as a basis. Meanwhile, each of the remaining $N - M$ eigenvectors are non-zero only in the components corresponding to one of the subgraphs, and so are localized. This $N - M$ dimensional subspace is orthogonal to the $M$ dimensional subspace corresponding to the zero eigenvalues.

Next, the impact of connecting the various subcomponents of the graph is determined. If the added lines are sufficiently weak, the eigenvalues of the graph connected to the zero eigenvalues of the disconnected graph will remain significantly smaller than the other eigenvalues of the system. One of the $M$ eigenvectors corresponding to these small eigenvalues is $\mathbf{1}$, and this vector is orthogonal to the $N - M$ first eigenvectors of the disconnected graph. As long as the eigenvalues corresponding to the remaining $M - 1$ eigenvectors are much smaller than the other eigenvalues, these eigenvectors will be nearly perpendicular to the $N - M$ eigenvectors of the disconnected graph. Furthermore, each of the $M$ vectors for the connected graph will have identical or nearly identical components corresponding to each subcomponent of the graph. Interestingly, the graph-based analysis demonstrates that slow eigenvector components will be nearly equal in each graph region but does not directly provide the component values of the eigenvectors in each region. Since the weighted Laplacian of this graph fits the form used in the slow coherency theory, the values of the slow eigenvector components are identical to those derived by analyzing the slow time scale response of a slow coherent system [2].

Many of the existing algorithms for slow coherency-based partitioning involve selecting reference generators with different slow eigenvector components and then associating other generators with these references using correlation of eigenvector components or linear decomposition in the reference generator directions, as summarized in [3]. Meanwhile, similar algorithms have been proposed heuristically for finding small cutsets in graphs [4, 5].

B. An Eigenvalue Sensitivity-Based Approach to Partitioning

The graph-based analysis of the weighted Laplacian of a network suggests that coherent regions of the graph can be identified by identifying and excluding weak links between partitions. Specifically, we would like to find the minimum sum of branch weights such that the removal of these branches drives the first $M$ eigenvalues of $K$ to 0. A first order approximation to the change in an eigenvalue due to the removal of a branch is given by the eigenvalue sensitivity formulas. DeMarco and Wassner formulate an algorithm for partitioning a graph by excluding branches based on the sensitivity of the Fiedler eigenvalue to the removal of lines in the graph [1]. The algorithm can be summarized in four steps:

1. The sensitivity of the Fiedler eigenvalue to a unit change in a branch weight, or $\frac{d\lambda_2}{dk_j}$ is calculated for all non-zero branches $k_j$. For a graph with unity node weights, $\frac{d\lambda_2}{dk_j} = (v_i^1 - v_j^1)^2$.
2. Lines are excluded from the graph in order of decreasing sensitivity until the graph has been subdivided into two partitions.
3. If any lines within one of the partitions have been removed, these lines are reintroduced in the graph.
4. The eigenvalues and eigenvectors are recalculated for the partition with the smallest eigenvalue, and the partitioning algorithm is repeated on this connected graph. The process is continued until $M$ partitions have been determined.

C. A Multiple Eigenvalue Sensitivity Approach to Partitioning

Assuming that we know the number $M$ of areas into which we intend to partition the network, and that such a decomposition will result in $M$ eigenvalues with value zero, it makes sense to generalize the sensitivity algorithm to simultaneously consider the $M$ smallest eigenvalues. In the multiple eigenvalue case, the sensitivity to a unit line removal can be calculated as

$$\sum_{l=0}^{M-1} \frac{d\lambda_l}{dk_{ij}} = - \sum_{l=0}^{M-1} (v_{il} - v_{lj})^2,$$

The algorithm is adjusted to calculate the required $M$ eigenvalues and eigenvectors. Next, the sum of the unit sensitivities of the first $M$ eigenvalues is calculated for each line, and the most sensitive lines are excluded until the graph is partitioned into $M$ components. Again, lines removed within a partition are reintroduced.

D. A General Approach to Partitioning through Line Exclusion

Although we have focused on partitioning graphs using first-order sensitivity approaches, a variety of different meth-
ods for identifying lines that lie between partitions can be postulated. Most generally, a function \( f(k_{ij}, v_{1i}, v_{1j}, \ldots v_{M-1j}) \) can be used to identify weak interconnections between portions of the graph. For example, the correlation function

\[
f(k_{ij}, v_{1i}, v_{1j}, \ldots v_{M-1j}) = \frac{\sum_{i=1}^{N-1} v_i k_{ij} v_j}{\left( \sum_{i=1}^{N-1} (v_i)^2 \right) \left( \sum_{i=1}^{N-1} (v_i k_{ij} v_j)^2 \right)}
\]

suggested in [3] for associating loads with reference generators can also be used to exclude lines between generators. A weak correlation between two connected nodes suggests that these nodes do not move coherently in the slow modes and so are weakly connected. We refer to line exclusion partitioning using this function as eigenvector component correlation-based partitioning.

The general algorithm for partitioning is based on calculating the function \( f(k_{ij}, v_{1i}, v_{1j}, \ldots v_{M-1j}) \) for each line in the graph and then excluding lines until the graph is disconnected into \( M \) components. The algorithm can be further improved by recalculating the eigenvalues and eigenvectors of the graph after each line removal, though these calculations may be computationally intensive for large graphs.

IV. SIMULATIONS AND OBSERVATIONS

In order to demonstrate the partitioning algorithms, we partition a randomly generated network. The impact of line removal on state variables and branch flows in network-based linear dynamical systems is demonstrated through simulations. These simulations suggest that the response of the network to a line removal is localized within graph partitions. Finally, we analytically relate the response of networks to a line removal with the partitioning algorithm and eigenvalue sensitivity concepts.

A. Simulation of the Developed Partitioning Algorithm

We apply the multiple eigenvalue sensitivity-based partitioning method and the eigenvector component correlation-based partitioning method to a randomly generated network in order to highlight some of the characteristics of these algorithms.

A plot of the randomly generated network is shown in Figure 1. In this network model, the nodes are uniformly distributed in the unit square. Nodes that are located within a certain distance are connected to each other, and the weight assigned to each branch is inversely proportional to the length of the branch. Additionally, we assume that all nodes have uniform weights. The smallest eigenvalues of the Laplacian are shown in Fig. 2. The significant jumps in values between \( \lambda_2 \) and \( \lambda_3 \), and between \( \lambda_5 \) and \( \lambda_6 \), indicates that this graph may be accurately partitioned into three or six sections. The network is partitioned into both three and six components using both of the partitioning algorithms (See Figures 3 and 4). The multiple eigenvalue sensitivity-based partitioning method and eigenvector component correlation-based partitioning method produce identical network partitions when the graph is subdivided into three pieces. In contrast, the two methods produce different partitions when the graph is subdivided into six components. In most of our simulations, the two methods have generated similar partitions, though the sensitivity-based method seems more likely to find small cutsets while the correlation-based method produces more evenly sized subgraphs. For this network, all four simulations originally excluded only one line within one of the components, suggesting that the algorithms are identifying weak links between components accurately.

B. Simulations of Line Removals

One possible application of network partitioning is the identification of branches and nodes that are affected by fault conditions in a given part of a network. If the developed partitioning scheme is useful in this context, we expect that perturbations in one of the partitions will largely be localized in that partition and perhaps in the lines leaving the partition. We simulate the response to line removals in linear systems defined on these networks. The simulations demonstrate that the impact of these line removals is localized in partitions as long as they occur within partitions. Furthermore, in linear dynamical systems, certain temporal characteristics of the response signals are also strongly correlated with the partitioning scheme.

We consider the three nominal systems which were pre-
The smallest eigenvalues of the Laplacian matrix.

where $\mathbf{x}$ is a vector of node variables and $\mathbf{u}$ is a vector of inputs at each node. We note that the nominal state vectors in these systems are only determined to within an additive constant at each instant in time, since the vector of all ones is in the null space of $\mathbf{K}$. We define the line flow between two nodes $i$ and $j$ as $k_{ij}(x_i - x_j)$. The nominal line flows are uniquely specified at each instant of time.

Next, we model a line removal in these linear systems as a rank one perturbation in the Laplacian matrix. Specifically, if the line between nodes $i$ and $j$ is broken, the weighted Laplacian changes to $\mathbf{K} = \mathbf{K} + \Delta \mathbf{K}$, where $\Delta \mathbf{K} = -k_{ij} \mathbf{z} \mathbf{z}^T$. The column vector $\mathbf{z}$ has a value of 1 in component $i$, a value of $-1$ in component $j$, and is zero otherwise. Because the modeled systems are linear, the differential effect of a line removal can be modeled as another system with the same Laplacian. The input to this differential system is non-zero only at the two nodes at the end of the removed branch. Because of superposition in linear systems, the input and output at these nodes is given by $\pm k_{ij}(x_i - x_j)$, or the nominal flow in the branch at the time of the line removal. We simulate the differential response of node variables and line flows to the removal of a line.

We first give an example of the differential response of the steady-state system $0 = -\mathbf{K}\mathbf{x} + \mathbf{u}$ to a line removal. In order to gauge the localization of the perturbation, we have calculated the maximum absolute line flow among all network branches and have plotted the branches that carry at least one-tenth of that maximum flow (Figure 5). We note that the absolute value of the maximum flow in a line due to this event is approximately 28% of the original line flow, which is a considerably decrease. Furthermore, the plots of lines carrying a fraction of the maximum flow suggest that the graph is accurately partitioned. The lines that carry one-tenth of the maximum flow are completely contained within the affected partition in the multiple eigenvalue sensitivity-based six-way partitioning of the graph. In fact, these lines are contained

Fig. 2. The smallest eigenvalues of the Laplacian matrix.

Fig. 3. The network is partitioned into three and six pieces using both the multiple eigenvalue sensitivity-based method and the eigenvector correlation-based method. These plots show the final partitions of the graph after lines that were excluded within a component have been reintroduced.
within a small portion of this partition, suggesting further localization.

Similarly, the differential response of (8) to a line removal is graphed (Figure 6). Specifically, we calculate the maximum of the absolute value of each line flow as a function of time. We then identify the line with the largest maximum flow and plot all lines that carry at least one-tenth of that maximum at some time. Again, the response is entirely contained within the partition containing the line removal. Plots of line currents inside and outside the affected partition also demonstrate that the transient response of the system is much faster in the affected partition than in the other partitions.

The differential response of (9) to a line removal is also localized (Figure 7). Again, the lines for which the maximum flow is at least one-tenth of the maximum flow in the network are plotted. Because the system is an undamped second-order system, the perturbations eventually become extensive, so that the whole network is affected. However, the response remains considerably smaller outside the partitioned region within the time frame of the simulation, approximately one cycle of the slowest mode. Also, a delay in the onset of the differential response for nodes and lines outside the perturbed region is observed.

C. Analysis of Line Removal Responses

The analysis of a line removal in power systems leads to an interesting relationship between partitioning methods and the impact of line removals in the system. Specifically, the magnitude of the steady-state differential response in state variables due to the removal of a line is closely connected with the line sensitivities that are used to partition the network. In this analysis, we assume that the state variables in the differential system response sum to zero; an offset in the state variables does not change the behavior of the system and so is unimportant. The following notation is used: \( \lambda_i, 0 \leq i \leq N - 1 \) are the eigenvalues of the weighted Laplacian \( \mathbf{K} \), \( \mathbf{v}_i \) are the corresponding eigenvectors, \( \mathbf{u} \) is the input to the differential system, and \( \mathbf{x} \) is the vector of state variables. A modal decomposition of the system demonstrates that

\[
\lambda_i \mathbf{v}_i' \mathbf{x} = \mathbf{v}_i' \mathbf{u} \quad \forall i.
\]
Consequently,
\[ v'_i x = \frac{1}{\lambda_i} v'_i u \quad \forall i \geq 1 \] (11)

while \( v'_0 x = 0 \) by assumption. Next, since the eigenvectors are orthonormal, the square of the magnitude of the response \( x \) can be calculated as
\[ x'^2 = \sum_{i=1}^{N-1} \frac{1}{\lambda_i^2} u'_i v'_i u. \] (12)

Because the input is based on a line removal, this expression can be rewritten as
\[ x'^2 = \sum_{i=1}^{N-1} \frac{1}{\lambda_i^2} (v_i - v_j)^2, \] (13)

where \( i \) and \( j \) are the nodes corresponding to the removed line. Next, we note that \( (v_i - v_j)^2 \) is the sensitivity of eigenvalue \( \lambda \) to a unit perturbation in a line between nodes \( i \) and \( j \), so that the magnitude of the voltage response can equivalently be written as
\[ x'^2 = \sum_{i=1}^{N-1} \frac{d\lambda_i}{d\lambda_i} \frac{1}{\lambda_i^2} (v_i - v_j)^2. \] (14)

The expression for the magnitude of the state vector response to a line removal is quite similar to the expression for determining which lines are excluded in the sensitivity-based partitioning method. The expression for the squared magnitude of the differential response suggests another possible partitioning function. Specifically, \( \sum_{i=1}^{N-1} \frac{1}{\lambda_i^2} (v_i - v_j)^2 \) can be calculated by removing each line in the system. The lines for which this sum is large should have extensive voltage responses, suggesting that they connect different partitions. Consequently, the line are removed in order of decreasing voltage response sizes until the correct number of partitions are found. To make the problem less numerically intensive, the sensitivities can be approximated from the eigenvectors of the weighted Laplacian of the full system rather than from the numerous reduced systems. Figure 8 shows the result of using the magnitude of the state vector perturbation to partition the example network. Specifically, the lines with extensive fault responses are removed from the system until the graph is subdivided into three and six components.

In partitioning the network using the magnitude of the state vector response, we have claimed that a large state vector response to a line removal implies an extensive response. Alternatively, we can argue that a small state vector response is localized. This is supported by the No-Gain Theorem developed in [6] that verifies that the maximum voltage due to a perturbation is non-increasing with a graph-based distance from the fault. Specifically, the nodes in the graph can be placed into tiers around the faulted pair of nodes. The No-Gain Theorem verifies that the largest voltage response due to
V. Conclusions

This paper has reported on our initial studies in partitioning electrical networks using graph theoretic techniques. We have examined the results of two partition methods. The first is based on the sensitivity of a number of small eigenvalues to line connection weights to identify those lines that will cause the system to break into separate groups. The second method uses correlations to identify weak connections. Neither method requires the identification of “reference” generators with which to group the remaining nodes.

A comparison of the two methods on a sample graph shows that for a small number partitions (three) the methods identify the same regions. For a larger set of partitions the areas vary somewhat. Simulation of a representative outage (line removal) initiated within an area results in dynamics that are essentially localized within the same area. Analyzing the results of the simulations leads to state response approach to partitioning. This state response method is analytically related to a eigenvalue sensitivity metric.

All results are encouraging because the simulations verify that localized responses can be expected, and all the partitioning methods give reasonable results. More research is required to further distinguish and evaluate the different methods. Part of the challenge is to determine a suitable metric for comparison. At this point we feel that the magnitude of the state vector response seems to be an appropriate metric, and partitioning based on it should be pursued further.

All of the discussion in this paper needs to be carried out using node weights, and incorporating load buses. The analysis with the inclusion of load buses is readily accomplished by use of generalized eigenvalue techniques. Non-uniform node weights are somewhat problematic in our approach because the resulting dynamic system matrix does not have a Laplacian structure. Nevertheless, a simple scaling of variables results in a symmetric system matrix, which one can analyze using techniques similar to those we have presented here.

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