Studies on the Probabilistic Collocation Method and its Application to Power System Analysis

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Abstract—PCM, originally applied to the analysis of power systems in [1], is a technique by which uncertainties in intensive power-system computations (such as load-flow solutions or transient dynamics) can be related to parametric uncertainties using only a small number of simulations. In this article, we identify and pursue two directions of study needed for broader application of PCM in power systems analysis. First, we generalize PCM to allow study of systems with multiple dependent uncertain parameters. Second, we develop a heuristic but systematic procedure for properly selecting the order of the PCM model that relates the uncertain parameters to the outputs. Third, we briefly explore approaches for interfacing PCM with other methods for evaluating uncertainties through simulation, such as boundary load flows; these meshed approaches may be required for systems with many uncertain parameters. In our development, we consider two small power systems examples—a load flow analysis for a system with uncertain loads and a transient simulation of a disturbance—to illustrate PCM’s potential applicability.

Index Terms—uncertainty analysis, power flow, probabilistic collocation, time-step simulations

I. INTRODUCTION

Intensive computation is needed for many studies of power systems, including for obtaining load flow solutions and for characterizing fast power system transients using time-step simulations. The probabilistic collocation method (PCM) recently has been advanced as a method for efficiently evaluating parametric uncertainties in time-step simulations [1][2], but holds promise more generally for achieving various computationally-intensive analyses under uncertainty. In this article, we identify and pursue several further directions of studies on PCM that are required for broader application of the method to power system analysis.

There is a wide literature on uncertainty analysis in power systems (see, e.g., the bibliography [3]). PCM contributes to power systems uncertainty analysis in that it allows efficient evaluation of uncertainties from pre-built simulation tools. We refer the reader to [1] for more details on the connections between PCM and other techniques for evaluating power system uncertainties.

Broadly, PCM (originally developed by researchers of global climate change [4]) is applicable to power systems analyses when these analyses are computationally costly, thus making evaluation of uncertainty through exhaustive Monte Carlo simulation difficult. PCM seeks to reduce this complexity by assuming a structured (polynomial) mapping between uncertain parameters and simulation results and identifying a good set of simulations for correctly and robustly determining the mapping. In this article, we aim to extend PCM in two respects:

1. Because power systems are often subject to multiple uncertainties that are in general correlated (e.g., weather dependent), we develop generalizations of PCM for systems with multiple correlated uncertainties.

2. To facilitate practical implementation of PCM, we develop a heuristic algorithm for checking whether a sufficiently high-order polynomial has been used to represent the mapping from parameter to simulation result. In this manner, we can avoid extraneous simulations of the system, and hence reduce computational cost.

We apply our new results concerning PCM to two small examples. First, a load flow solution for a 5-bus model with two correlated uncertain loads is used to illustrate our generalization of PCM. Second, we consider transient analysis of a system that suffers a disturbance to a feeder line (see [5]), in which the uncertain parameters are the recovery time constant for a load and the tap-changing interval for a transformer. Our study of the load flow example also motivates a new approach for evaluating uncertainties in simulations, that meshes PCM with an optimization algorithm such as the boundary load flow (e.g., [6]).

The remainder of the article is organized as follows. In Section II, we summarize the probabilistic collocation method. Section III describes our generalization of PCM to systems with multiple correlated uncertainties, and is illustrated using the load-flow example. In Section IV, we present our algorithm for selecting the proper order in PCM. Finally, in Section V, we consider application of PCM to the dynamic simulation of a power system disturbance.

II. REVIEW OF THE PROBABILISTIC COLLOCATION METHOD

The probabilistic collocation method is a means for developing a parametric model for the deterministic mapping
between a stochastic input and an output (Figure 1), using only a small number of simulations of the system. In particular, **nth-order PCM** seeks to represent the mapping using an nth-degree polynomial, whose coefficients are found by matching the model predictions with simulation outputs for a particular set of n+1 input values. The n+1 input values---henceforth called the **PCM Collocation Points**---are specially chosen, in a manner that makes the fit robust to some possible errors in the model’s parameterization. Specifically, the n+1 PCM Collocation Points are chosen so that the mean output predicted by the model is identical to the actual mean output, if in fact the mapping is a polynomial of any degree less than or equal to 2n+1. Thus, PCM specifies a low-order mapping that approximates a much higher-order (in other words, more detailed) mapping, in the sense that the mean output predicted by both mappings is identical.

![Stochastic input x with distribution f(x) → Deterministic Simulation/Mapping g() → Stochastic Output g(x)](image1)

**Stochastic input x**

![Stochastic input y](image2)

**Stochastic input y**

![x and y have joint distribution f(x,y)](image3)

![Figure 1: Mappings with one and two stochastic inputs, respectively, are shown. PCM can be used to characterize the mapping g() and the probability distribution of the output with a small number of simulations.](image4)

It is worthwhile for us to briefly describe the process for choosing PCM Collocation Points, which draws on the theory of **Gaussian quadrature** (see, e.g., [7]). In particular, we consider a system or mapping with scalar stochastic input x, which is distributed according to a probability density function f(x) that is non-zero over the finite continuous domain A. We seek to approximate the functional mapping g(x), that transforms the input to the output. Notice that the mean value of the output in this case is given by

$$E(x) = \int_A f(x)g(x)dx.$$  

(1)

Gaussian quadrature allows us to choose n+1 points $x_1, \ldots, x_{n+1}$ such that, for any $g^*(x)$ that is a polynomial of order less than or equal to 2n+1 and for which $g^*(x_i) = g(x_i), i = 1, \ldots, n+1,$ the integral $\int_A f(x)g^*(x)dx$ is the same. Thus, the mean value predicted by the degree-(n+1) polynomial that passes through these points is the same as the mean predicted by any polynomial of degree less than or equal to 2n+1 that passes through the points. Equivalently, the degree-(n+1) polynomial suffices to capture the mean output, if the mapping is indeed a polynomial of degree less than or equal to 2n+1. The Gaussian quadrature points (in our case PCM Collocation Points) are determined by computing the first n+1 orthogonal polynomials with respect to f(x) (see [7]); the roots of the degree-n+1 orthogonal polynomial are the PCM Collocation Points. We refer the reader to [2] for details on the selection of PCM Collocation Points and justification of quadrature. For our purposes, it suffices to say that a systematic algorithm for determining orthogonal polynomials and hence for selecting PCM Collocation Points is well-known, and that the computational effort required for PCM is small compared to the cost of simulation in many realistic power system examples.

In concluding our review, we note that PCM has largely been advanced as a heuristic method, in that the low-order fit matches higher-order fits only in an average sense and that only if the higher-order fit is in fact a polynomial. However, we mention that there is a wealth of theoretical work on Gaussian quadrature that can be applied to construct error bounds on the predicted output statistics, and on the predicted mapping (see, e.g., [7] and [8]). Also, previous studies concentrating power systems [1] and global climate modeling [4] demonstrate that PCM can provide good approximations for mappings in an efficient manner.

**III. MULTIPLE CORRELATED INPUTS: CONDITIONAL PCM**

In the analysis of power systems, as in other contexts, we may often need to characterize systems with multiple correlated uncertain inputs. For instance, the load flow solution of a realistic system is usually dependent on multiple loads, which are realistically modeled as uncertain but correlated. Monte Carlo simulation of systems with multiple uncertain parameters is often especially taxing, because the number of simulation points needed to identify the mapping between the inputs and the output grows exponentially with the number of inputs. Thus, it is valuable to develop techniques that can be used to identify mappings between multiple inputs and an output with sparse simulation. We extend PCM to represent such multi-dimensional mappings.

For convenience, let us first discuss our generalization of PCM to systems with two correlated, uncertain inputs (see Figure 1). We call this generalization **two-dimensional PCM**. We assume that the two uncertain inputs x and y are jointly distributed according to a density function f(x,y) that is non-zero over a finite, convex two-dimensional domain A. Our aim is to identify the mapping $g(x,y)$ that specifies the output in terms of these inputs. We assume that this mapping can be approximated by a bivariate polynomial of the form

$$g^*(x, y) = \sum_{i=0}^{n} \sum_{j=0}^{n} a_{ij} x^i y^j.$$  

(2)

Henceforth, we refer to $g^*(x,y)$ as a **generalized polynomial of degree n** (see, e.g., [9] for a discussion of polynomials with multiple variables). We feel that a generalized polynomial representation for a two-dimensional mapping is appropriate, because (as in the one-dimensional case) higher-degree generalized polynomials provide more and more detailed
representations of the mapping. More specifically, an order-$n$ generalized polynomial representation allows us to specify a set of polynomial mappings between each single input and the output, given the other input. To determine the coefficients in (2), we simulate the output for a particular set of $(n+1)^2$ input pairs, which we again call PCM Collocation Points. From the $(n+1)^2$ outputs at the PCM Collocation Points, we determine the $(n+1)^2$ coefficients by solving a system of linear equations. As in the one-dimensional case, the success of two-dimensional PCM depends strongly on appropriate choice of the PCM Collocation Points. We propose the following algorithm for choosing the PCM Collocation Points:

1. We compute the marginal distribution for the input $x$ as $f(x) = \int_A f(x, y)dy$. We then find the degree-$(n+1)$ orthogonal polynomial with respect to $f(x)$, and find the roots of this polynomial. Notice that these are the $x$ values that we would choose as PCM Collocation Points, if we were applying one-dimensional PCM of order $n$ to find a mapping between $x$ and an output. Let us label these points $x_1, \ldots, x_{n+1}$.

2. We compute the conditional distributions $f(y \mid x_i) = \frac{f(x_i, y)}{f(x_i)}$. We then find the degree-$(n+1)$ orthogonal polynomials with respect to each distribution, and find the roots of these polynomials. Let us call the roots of the orthogonal polynomial with respect to $f(y \mid x_i)$ as $y_i(x_1), \ldots, y_{n+1}(x_i)$.

3. We use the $(n+1)^2$ pairs of inputs $[x_i, y_j(x_i)]$, $1 \leq i \leq n+1$, $1 \leq j \leq n+1$, as the PCM Collocation Points.

The following analytical results (presented without proof) can be deduced for two-dimensional PCM; these results motivate use of the method:

1. Given that the input $x$ is any one of the values $x_1, \ldots, x_{n+1}$, the mean output is correctly predicted by two-dimensional PCM whenever the actual mapping is a generalized polynomial of degree less than or equal to $2n+1$. Also, from continuity arguments, we can argue that the mean output predicted by PCM is nearly correct for inputs $x$ that are close to one of the points $x_1, \ldots, x_{n+1}$. Since the points $x_1, \ldots, x_{n+1}$ are chosen to reflect the high-probability domain for the input $x$ (this is one of the benefits of one-dimensional PCM), two-dimensional PCM predicts the mean output correctly given likely values for $x$.

2. In the special case that $x$ and $y$ are in fact independent, the (unconditioned) mean value for the output is correctly predicted by PCM whenever the mapping is a generalized polynomial of degree less than or equal to $2n+1$. Further, in the more general case that $x$ and $y$ are not independent but the $r$th-

conditional moment for $y$ given $x$ is an $r$th-order polynomial, PCM predicts the output mean whenever the actual mapping is a true two-dimensional polynomial of degree less than or equal to $2n+1$ (i.e., a sum of monomial terms, each of which has total degree less than or equal to $2n+1$).

3. The PCM Collocation Points always fall within the region $A$, so that we are always able to simulate a meaningful output for each PCM point.

We note that PCM can easily be generalized to identify mappings between three or more uncertain inputs and an output. As in two-dimensional PCM, we can select PCM Collocation Points for higher-dimensional PCM recursively from a sequence of marginal and conditional distributions. These higher-dimensional PCM algorithms are amenable to the same analyses as two-dimensional PCM; we do not consider large examples in this study and so omit the details.

In the remainder of this section, we apply two-dimensional PCM to characterize the mapping between the two uncertain loads and the load flow voltage at a bus in a power system. Our study is in the context of a stylized example obtained from [10], and is not meant to provide a comprehensive depiction of load flow uncertainties by any means. Our primary purpose is to illustrate two-dimensional PCM, and to explore some potential benefits and caveats of using PCM to characterize load flow solutions.

PCM-based characterization of load flow voltages falls within the broad class of probabilistic load flow (PLF) algorithms (see [11] for a summary of some work on PLF). These are methods for computing uncertainties on load flow solution parameters (e.g., bus voltages or line loadings), given uncertainty distributions on load powers and other system parameters). A full study of the literature on PLF is beyond the scope of this article, but we present a few general concepts. As discussed in, e.g., [6] and [11], PLF algorithms are either based on Monte Carlo simulation techniques, on exact analysis, or on some combination of these. Very often, analytical methods assume a load flow model that is linearized around one or multiple equilibria, and require some structure (e.g., Gaussianity) in the parameter distributions. Monte Carlo techniques account for the nonlinearities in the load flow solution and allow for general input parameter distributions, but are computationally intensive. As an alternative to PLF, load flows for systems with uncertain inputs have also been characterized by identifying limits on the output variables given limits or distributions on the inputs (e.g., [6], [12]). These methods, called boundary load flow algorithms, have recently been combined with techniques that provide fuzzy-set descriptions of output variables, given fuzzy descriptions of input variables [6]. We believe that PCM can contribute to PLF analysis, by providing an simulation strategy with low computational cost and also by providing a method for meshing probabilistic and boundary methods.

We apply PCM to find the PLF solution in the small power system example shown in Figure 2. In this example, we assume that the scaling parameters (inputs) $x$ and $y$ (see Figure 2), which specify the load power magnitudes as buses 4 and 5,
are jointly distributed as shown in Figure 3. The positive correlation is meant to reflect that load requirements tend to be correlated with external parameters (e.g., temperature), which are roughly constant over a set of loads. Our output variable is the magnitude of the voltage at bus 4.

Figure 2: We apply PCM to characterize the voltage at bus 4, given that the loads at buses 4 and 5 are uncertain.

Figure 3: The parameters (inputs) $x$ and $y$ are distributed uniformly over the polygonal region shown. The PCM Collocation Points are also illustrated.

Application of PCM to this example first requires computation of the PCM Collocation Points; the nine points for second-order PCM are shown on Figure 3. Using the PCM Collocation Points, we characterize the mapping between the inputs and output. The second-order generalized polynomial representation for the mapping found using PCM is the following:

$$g^*(x, y) = -0.041x^2y^2 + 0.20x^2y - 0.24x^2$$
$$+ 0.12xy^2 - 0.61xy + 0.72x - 0.10y^2 + 0.45y + 0.48.$$
This predicted mapping is compared to a mapping generated through exhaustive simulation in Figure 4. We note that the PCM prediction, which requires only nine simulation points, is essentially indistinguishable from the mapping generated through brute-force simulation, which we construct using 400 simulations. Thus, our example highlights the significant computational savings that can be obtained through use of PCM. Finally, we numerically determine the distribution for the output variable and compare it to the actual output distribution in Figure 5.

For this simple example, PCM characterizes both the input-output mapping and the output distribution well. Our solution highlights that the mapping between input and output over the domain of the uncertain loads is non-linear, especially because the correlation between the two loads makes heavy loading conditions frequent. PCM is able to capture this non-linearity, while (in this example) requiring only nine carefully-chosen simulation points to develop a good quadratic mapping. This ability to capture non-linear mappings using only a small number of simulations suggests that PCM holds promise as a PLF algorithm. We note that PCM is also advantageous in this example, in that we could allow uncertainties with arbitrary joint distributions on the input parameters.

One difficulty in applying PCM is that the number of required PCM Collocation Points typically grows exponentially in the number of uncertain parameters. When the number of uncertain parameters becomes large, we note that meshing PCM with a boundary load flow algorithm can provide a tractable solution. In particular, we can select PCM Collocation Points for a few significant or important uncertain parameters; for each PCM point, we can apply a boundary load flow algorithm with respect to the other uncertain parameters, to find the largest and smallest possible output. Using these extremal outputs, we can develop a pair of mappings from the significant inputs to the output using PCM, which serve as bounds on the actual mapping. Such a meshed algorithm is best illustrated with an example. A plausible alternate description for the load scaling parameters in Figure 2 is that these parameters have a strong dependence on a single uncertain input parameter (e.g., temperature) with small, independent deviations from this predicted dependence. For instance, the two parameters could have the form $x = T + \epsilon_1$ and $y = T + \epsilon_2$, where $T$ is a significant random parameter, and $\epsilon_1$ and $\epsilon_2$ are small, independent random parameters.

While we could apply three-dimensional PCM to such a system, a less computationally intensive approach is the following. We can choose PCM Collocation Points as if we are applying one-dimensional PCM, in which the uncertain parameter is $T$; for each of these PCM Collocation Points, we can compute the extremal output values over the domain of $\epsilon_1$ and $\epsilon_2$ (see [6] for an efficient means for doing so). We can then develop one-dimensional polynomial representations for both sets of extrema. An example of such “boundary mappings” is shown in Figure 6.

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IV. ORDER SELECTION IN PCM

Practical application of PCM requires proper choice of the PCM order (i.e., of the degree of the polynomial or generalized polynomial used to fit the input-output mapping). Because application of PCM requires a new set of simulations for each order considered, a systematic technique for deciding the appropriate PCM order without excessive simulation is needed. Our studies of one-dimensional PCM suggest the following three-step heuristic algorithm:

1. We apply PCM of successive orders (beginning with first-order PCM), until visual inspection suggests that the predicted mapping has not changed between two successive applications.
2. If the mapping predicted by the second-highest-order PCM applied in step 1 has at least two extrema, the visually-determined PCM fit is in our experience the proper one. (When the mapping has several extrema, we find that the PCM fit converges dramatically to the correct mapping beyond a certain order, so that visual inspection is sufficient to identify the proper fit.) Order-selection is illustrated for a mapping with three extrema in Figure 7.
3. If the second-to-last PCM prediction from the first step has fewer than two extrema, we require an
analytical comparison measure to determine whether or not a sufficient order has been chosen. In particular, we numerically compute the output distribution using the mapping of each order. We then compute the Kullback-Leibler (KL) distance between successive pairs of distributions (see [13]); if the KL distance between the highest two-order PCM output distributions is sufficiently small (i.e., drastically smaller than the KL distances between lower-order fits), then sufficiently high-order PCM has been used. Otherwise, a higher-order PCM algorithm should be applied, until a sufficiently small KL distance is obtained. We note that, if we desire a completely automatic algorithm for order-selection, we can use comparisons of KL distances regardless of the number of extrema.

We have recently considered application of the order-selection algorithm for PCM in higher dimensions. Our preliminary studies suggest that the algorithm described above is applicable in higher dimensions also: order can be selected by visual inspection when the mapping has multiple extrema, and otherwise can be inferred using the KL distance. We have applied order selection algorithm to the load flow example developed above, and found that a second-order generalized polynomial indeed provides a good fit.

![Comparison of Plots of PCM generated Polynomials](image)

Figure 7: We illustrate PCM order selection through visual inspection, for a mapping that has three extrema.

<table>
<thead>
<tr>
<th>PDF Comparison</th>
<th>KL Distance</th>
</tr>
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<tbody>
<tr>
<td>PCM 2^d Vs. PCM 3^d</td>
<td>0.1332</td>
</tr>
<tr>
<td>PCM 3^d Vs. PCM 4^th</td>
<td>0.1134</td>
</tr>
<tr>
<td>PCM 4^th Vs. PCM 5^th</td>
<td>0.0977</td>
</tr>
<tr>
<td>PCM 5^th Vs. PCM 6^th</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

V. EXAMPLE: DYNAMIC SIMULATION OF A DISTURBANCE

In the context of power systems, PCM was originally advanced as a tool for evaluating uncertainties in time-step simulations of transient dynamics ([1] and [2]). PCM is potentially valuable for evaluation of uncertainties in transients, because it can reduce the number of simulations (which are very often computationally intensive) required for uncertainty analysis. PCM also has the advantage that it can be implemented without significant modification of the time-step simulation programs for transients, since it only requires measurement of output values for various inputs.

Here, we apply two-dimensional PCM to characterize a small power system’s transient response to a disturbance. The example that we use is drawn from [5], where it is also used to illustrate characterization of transient-simulation uncertainties, using trajectory-sensitivity methods. Our explorations of this example illustrate how PCM compares with, and complements, the trajectory-sensitivity based methods.

![Dynamic Simulation of a Disturbance](image)

Figure 9: We consider the response of this power system to a disturbance, in particular tripping of the line with admittance X1. The uncertain parameters in this example are the load recovery time constant and the tap-changing interval of the transformer.

The small system shown in Figure 9 is disturbed through tripping of one of the lines between the supply point and bus 1. We consider the transient response of the voltage magnitude at bus 3. This transient response is modulated by the recovery dynamics of the load, as well as the logic of the tap-changing transformer. It is in the parameters of these recovery dynamics that we assume some uncertainty (in accordance with [5]). In particular, we assume that the load time constant $T_p$ and the interval between tap changes $T_{up}$ are uniform and independently distributed, over the intervals [3,7] and [15,25], respectively.

We apply PCM to characterize the mapping between the inputs $T_p$ and $T_{up}$ and an output of interest, which we choose to be the minimum voltage on bus 3 during the duration of the simulation. We find that a second-order generalized polynomial model is sufficient to specify the mapping (Figure 10). Thus, with only nine simulations, we are able to extract the mapping between the inputs and the output, and further to expose that this mapping is not linear. We refer the reader to Appendix B for a details on the PCM collocation points and the obtained mapping.

A compelling feature of PCM is that, using the nine selected simulations, we can in fact characterize many different output features (e.g., the output voltage at specific times, or various flows in the power network). We note that our analysis compares favorably with the trajectory sensitivity analysis in that we simulate the actual power system rather than a linear approximation thereof. We caution, however, that each
simulation of the actual power system may be very expensive compared to a trajectory sensitivity-based simulation; it is only because so few points are required for PCM that our analysis is feasible. Finally, we mention that one further possible application of PCM to power system dynamic simulations is to identify whether linear relationships between input and output variables hold, and hence to evaluate whether trajectory sensitivity analyses can be used.

![Minimum Voltage During Transient](image)

Figure 10: The PCM-generated mapping between two uncertain parameters and the minimum voltage reached at Bus 3 during a transient simulation is shown.

VI. CONCLUSIONS

In this article, we have explored the application of the Probabilistic Collocation Method (PCM) to the characterization of uncertainties in power system computations. To this end, we have extended PCM in two ways, namely to allow study of systems with multiple uncertain parameters and to automatize the order-selection process. Using these extensions, we have applied PCM to evaluate the effect of load power uncertainties in a power flow computation, and to characterize the dynamic response of a power system to a disturbance when certain recovery parameters are uncertain.

Our studies show that PCM holds great promise as a tool for uncertainty evaluation in power systems, because it allows identification of mappings between uncertain parameters and outputs of interest using simulations for only a few sets of parameter values. In the examples that we have considered, PCM has been quite effective in providing a sparse set of simulation points, from which the mapping between parameters and output can be identified well over the domain of likely parameter values. Although our studies are preliminary and application of PCM to larger power system examples has yet to be considered, our results suggest that PCM is a compelling approach to uncertainty evaluation for computationally-intensive power system analyses.

REFERENCES


APPENDIX A: DETAILS OF THE LOAD FLOW EXAMPLE

We list the details of the load flow example considered in Section III:

- Admittance Matrix:

\[
\begin{bmatrix}
2 - j20 & -1 + j10 & 0 & -1 + j10 & 0 \\
-1 + j10 & 3 - j30 & -1 + j10 & -1 + j10 & 0 \\
0 & -1 + j10 & 2 - j20 & 0 & -1 + j10 \\
-1 + j10 & -1 + j10 & 0 & 3 - j30 & -1 + j10 \\
0 & 0 & -1 + j10 & -1 + j10 & 2 - j20 \\
\end{bmatrix}
\]

- Bus 1 is the slack bus.
- Bus 2 is a PV bus, with injected active power 0.883, voltage magnitude 1.
- Bus 3 is a PV bus, with injected active power 0.0076, voltage magnitude 1.
- Bus 4 is a PQ bus, with power extraction \( x(1.71 + j0.598) - j \).
- Bus 5 is a PQ bus, with power extraction \( y(1.73 + j0.550) - 0.8j \).
APPENDIX B: DETAILS OF THE TRANSIENT SIMULATION STUDY

We detail the transient simulation study considered in Section V. The line parameters (see Figure 9) are

\[ X_1 = 1.8, X_2 = 3, X_3 = 1.5 \]

and the nominal power required by the load is \( P_d = 0.5 \). The disturbance is the opening of Line 1 (i.e., the line with admittance \( jX_1 \)). Upon loss of the line, the load exhibits dynamic recovery, and the tap-changing transformer is used to re-establish the bus voltages. We refer the reader to [5] for details on the load recovery dynamics and tap-changing logic.

Nine simulations are required to fit the mapping between the two inputs and output with a second-order generalized polynomial using PCM. The PCM collocation points turn out to be the following coordinate pairs \((T_p, T_{tap})\): \((3.45,16.2)\), \((3.45,20)\), \((3.45,23.8)\), \((5,16.2)\), \((5,20)\), \((5,23.8)\), \((6.55,16.2)\), \((6.55,20)\), \((6.55,23.8)\). The second-order generalized polynomial for the mapping between the inputs and output obtained using PCM is

\[
g^*(x, y) = 0.90 + 10^{-3} (-0.0044x^2y^2 + 0.071x^2y \\
-0.011x^2 + 0.037xy^2 + -0.44xy + 1.6x - 0.019y^2 - 2.6y)\]