Homework 5

Cpt S 317, Spring 2017

Due Date: March 29, 2017

Total points: 48

1. (5 points)

   Let L be the set of all strings with an equal number of a's and b's. A CFG for L is as follows:

   $$G: \quad S \rightarrow aSbS \mid bSaS \mid \epsilon$$

   Give parse trees for deriving the strings $aaabbb$ and $baabbaba$ from G.

2. (6 points) (from Exercise 5.1.7 in book):

   Consider the CFG G defined by the productions:

   $$S \rightarrow aS|Sb|a|b$$

   Prove by induction that no string in L(G) has ba as a substring. Hint: To show this, you will have to do induction on the length of the string, for strings that be generated by the grammar.

3. (12 points)

   Give simple English language descriptions for the following grammars:

   a) $G_1: \quad S \rightarrow aS \mid a$

   b) $G_2: \quad S \rightarrow aSa \mid aa \mid a$

   c) $G_3: \quad S \rightarrow SaS \mid a$

   d) $G_4: \quad S \rightarrow aSa \mid aTb \mid bTa \mid bTb$

   $\qquad\quad T \rightarrow aT \mid bT \mid \epsilon$

4. (5 points) Is the following grammar ambiguous? Justify your answer.

$$G: \quad S \to SaS \mid a \mid b$$

5. (5 points) The following grammar generates *postfix expressions* with operands x and y and binary operators $+$, $-$, $*$ and $/$:

$$G: \quad E \to EE+ \mid EE- \mid EE* \mid EE/ \mid x \mid y$$

For the string $xyxyx - * + /$, give a parse tree, a leftmost derivation and a rightmost derivation.

6. (5 points) Give a *right-linear CFG* for the language of all strings that are of the form a^i where $(i \mod 3 = 2)$. Note that the alphabet here is $\{a\}$.

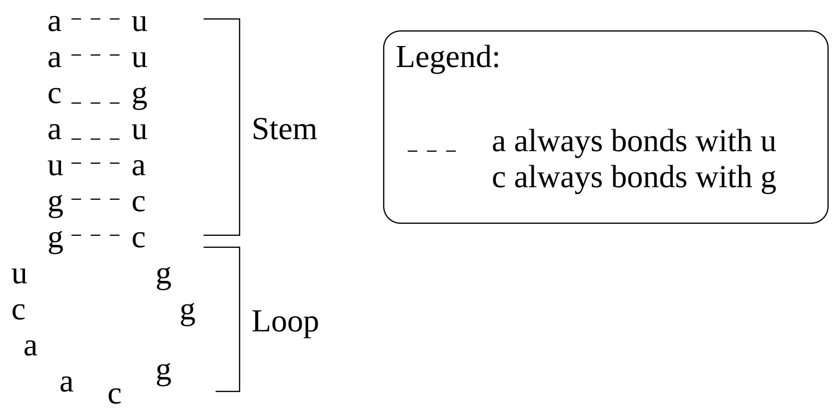
7. (10 points)

In the natural world, there are some instances where biological molecules follow rules like in a CFG. Consider the following example:

Cells have RNA molecules which can be represented as strings over alphabet $\{a, c, g, u\}$. An RNA molecule ($\equiv$"RNA string") w is said to have a *stem-loop* structure if w can be broken down into three parts: $w = xyz$, s.t., string x is a "reverse complement" of string z, and $|x| \geq 1, |y| \geq 1, |z| \geq 1$. "Reverse complement" of any string x is equal to another string z if z is obtained by first reversing x and replacing every a with u, u with a, c with g, and g with c.

For example, an RNA molecule w that reads *aacauggucaacgggccauguu* has a stem-loop structure, because there exists a combination x, y, z where $x = aacaugg$, $y = ucaacggg$, and $z = ccauguu$. Note that x's reverse complement is z. The reason why this is called "Stem-Loop" structure should be clear from the figure below, which shows the string bonding with itself in 2D (a prefix to a suffix) as defined by the reverse complement property.

RNA Stem-Loop for the RNA *aacauggucaacgggccauguu*:



a) Write a grammar for recognizing the language of RNA molecules (i.e., the set of strings over alphabet {a,c,g,u} that have a stem-loop structure).

b) Is your grammar ambiguous? If yes, give an example string with more than derivation. If not, just briefly justify what makes it unambiguous.