Case 1 – Real-Valued Poles. Use partial fraction expansion to find the inverse Laplace transform of each of the following. For each case, find all finite poles and zeros and plot their location in the $s$-plane. Qualitatively, how does the zero location affect the time response?

1. $X_1(s) = \frac{8}{s^2 + 12s + 20} = \frac{8}{(s + 2)(s + 10)}$.

2. $X_2(s) = \frac{8(s + 3)}{(s + 2)(s + 10)}$

3. $X_3(s) = \frac{8(s + 9)}{(s + 2)(s + 10)}$

4. $X_4(s) = \frac{8(s + 2.1)}{(s + 2)(s + 10)}$
Solution
1) Using partial fraction expansion,

\[ X_1(s) = \frac{8}{(s + 2)(s + 10)} = \frac{k_1}{s + 2} + \frac{k_2}{s + 10} \]. Evaluating,

\[ k_1 = (s + 2)X_1(s) \big|_{s=-2} = \frac{8}{(s + 10) \big|_{s=-2}} = 1, \]

\[ k_2 = (s + 10)X_1(s) \big|_{s=-10} = \frac{8}{(s + 2) \big|_{s=-10}} = -1. \]

Hence, \( x_1(t) = k_1 e^{-2t} + k_2 e^{-10t}, \) for \( t \geq 0. \)

Matlab code and plot follow.

\[
\begin{align*}
\text{>> t} & = \text{[0:0.01:3];} \quad \% \text{Time range [0, 3] sec}
\text{>> k1} & = 1; \text{k2} = -1;
\text{>> p1} & = -2; \text{p2} = -10; \quad \% \text{Specify poles}
\text{>> x1} & = \text{k1*exp(p1*t) + k2*exp(p2*t);} \quad \% \text{Inverse Laplace transform}
\text{>> plot(t,x1)}
\text{>> title('Inverse Laplace Transform of X_1(s)')} \\
\text{>> xlabel('Time, t, sec')} \quad \% \text{Time Signal x_1(t)}\end{align*}
\]
For Case 1, problems 2) – 4), plot of $x(t)$ for various zero locations.
Case 2 – Complex-Valued Poles. Use partial fraction expansion to find the inverse Laplace transform of each of the following. For each case, find all finite poles and zeros and plot their location in the $s$-plane. Use Matlab to plot the time functions.

5. $X_5(s) = \frac{s + 1}{s^2 + 2s + 2}$

6. $X_6(s) = \frac{1}{s^2 + 2s + 2}$

Case 3 – Repeated poles. Use partial fraction expansion to find the inverse Laplace transform of each of the following. For each case, find all finite poles and zeros and plot their location in the $s$-plane. Use Matlab to plot the time functions.

7. $X_7(s) = \frac{3s^2 + 3s + 1}{s(s + 1)^2}$